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International transmissions of income and growth

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INTERNATIONAL TRANSMISSIONS OF

INCOME AND GROWTH

by

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INTRODUCTION

The purpose of this essay is to present a systematic discussion of international transmissions of income and growth from the standpoint of pure theory.

The general background of the discussion of this topic in the literature is essentially the Keynesian analysis of income determination. It is then easy to draw the limitations of the conclusions arrived at, however the specific models employed may differ from each other. The short-run character and the underemployment setting of Keynesian analysis is carried over in considering the interactions of the incomes and their components of various countries linked together into an international economic system. The limited scope of this approach can bearly be concealed by the dynamization of the basic analytical tools of the static theory. Growth theory based essentially on dynamic IS and LM curves in highly aggregative models turns out to being no more than investigation of intertemporal equilibrium conditions. Wherever these conditions are not met, growth theory yields mostly embarrassing implications and certainly cannot be used for predictions. It is not however entirely useless. Economic theory cannot do away with a number of ideas which are verifiable under ideal conditions only, without reduction in the wealth of its "language". As it stands, this particular kind of theorizing gives much insight into the economic mechanism and at any rate it established the conceptual background for very recent attempts to adjust growth theory to empirical needs by disaggregation of the concept of national income and its large components, consumption and investment, into meaningful smaller aggregates.

This essay follows the pattern of the highly aggregative models and focuses its interest on the function of the foreign sector as a component of national income (exports) and transmitter of income and growth through the import function. The examination of the equilibrium conditions of a two-country international system in a dynamic context was done independently of Harry Johnson's contribution Johnson (15) but it was not carried as far, because of the complexity of the mathematical argument.

At the end of this essay empirical findings by researchers who employed essentially similar techniques in investigating international income transmissions have been summarized Neisser (27), Polak (30), Beckerman (4).

A few words should be said about price effects not being analyzed. The assumption of the existence of substantial underemployment which makes it possible to abstract from price effects cannot be considered to reflect the economic reality of the postwar period, in general. Hence, prices should be considered in any attempt at a more complete theory, especially when no empirical research (where it is very often the case that price effects are not statistically very significant for small disturbances) is undertaken.

The narrowness of the scope of the present work, explicitly recognized, it is hoped, serves as an excuse for the omission.

A MODEL OF AN OPEN ECONOMY

It is intended to first set up a model of an open economy, the unit in an international economic system. The orientation of the analysis at the beginning is to determine the equilibrium level of income of an economy engaged in free trade in a setting of perfect competition, where the aggregate production function is homogeneous of first degree (constant returns to scale). We shall further assume a constant price level domestically and internationally. The realism of the assumption about constancy of the price level depends on presupposing a general underemployment of resources and a perfectly elastic supply of them. Under these conditions a change in income does not induce price changes, at least over a considerable range in the path towards full employment. It is however an oversimplification to assume, as it is often the case, that the supply function of output is perfectly elastic up to a point in the neighborhood of full employment and perfectly inelastic beyond this point.

Keynes himself noted Keynes (16) that some price increases may occur at an earlier stage reflecting bottlenecks as long as excess capacity diminishes and all sorts of rigidities such as immobilities of resources, increasing difficulties in factor substitution, and effects of monopolistic groups become operative. Nevertheless, on the basis of empirical evidence there has been a tendency in the field of international trade to yield some priority to the income effects over the price effects, on the ground that the income elasticity with respect to effective demand is larger than the price elasticity.

A three-sector economy will be considered. The sectors contributing to the formation of the national income will be the consumption sector, the investment sector, and the foreign trade sector.

By taking account of a foreign sector, the following facts are acknowledged. The goods and services produced in a country are neither consumed nor used for investments entirely by residents of that country. A part of the goods and services domestically produced is exported. On the other hand, part of the aggregate national consumption consists of imported goods and services and part of the total amount of investment represents foreign investment into the country under consideration.

Finally, a country pays interest and dividends on capital borrowed from abroad and receives payments on interest and dividends resulting from capital which was lent by her to other countries. National income, then, is equal to: aggregate consumption, plus total net investment, plus the state of the balance of payments, or

$$Y = C + I + X - M$$

where Y stands for national income, M for imports of goods and services, and payments of dividends and interest abroad - henceforth it will be referred to as "imports" - C for aggregate consumption, I for total net investment, in other words total domestic plus net foreign investment and X for export of goods and services and receipts of payments from abroad on dividends and interest - which will be referred to as "exports".

It follows, that since in a closed economy $Y - C = S$; $I = S$ is, the equilibrium condition; while in an open system $S = I + X - M$, or $S - I = X - M$ where S = saving. This naturally means that in equilibrium the difference between saving and investment is matched by the difference of the balance of payments.

In the traditional way, the aggregate consumption will be expressed, in real terms, as a function of the national income.

Linear case		General case	
$C = cY + C_o$	(1a)	$C = C(Y)$	(1')

Where Y is national income,

$$c = C'(Y) = \frac{dC}{dY}$$

the marginal propensity to consume and C_o the component of consumption not dependent on Y. The marginal propensity to consume has been established in theoretical and empirical analysis as a parameter of considerable constancy.

If the model is to be not of the shortest run, an investment function must be introduced to relate investment expenditure to Y.

Then:

Linear case		General case	
$I = bY + I_o$	(2)	$I = I(Y)$	(2')

Where I_o is autonomous investment and b the fraction of Y spent on investment projects. Here one must state explicitly that Equations 2, 2' do not incorporate any established behavioral hypothesis regarding investment spending. It seems however, on the basis of research done up to date, that there is not any other functional relationship which explains investment in a satisfactory manner. Equation 2 then may be tentatively accepted as an approximation to the "true" but unknown functional form.

The foreign sector of the economy will consist of an import function and an export function.

The import function will express the value or volume of imports as dependent on the level of income.

Linear case

$$M = mY + M_o \qquad (3)$$

General case

$$M = M(Y) \qquad (3')$$

Where

$$M = M'(Y)\ \frac{dM}{dY}$$

is the marginal propensity to import and M_o is the autonomous component.

The import function is assumed in general to be monotonically increasing. Exports are given exogenously.

Linear case

$$X = \overline{X} \qquad (4)$$

On the basis of the above equations, the income-determination expression is:

Linear case

$$Y = cY + C_o + bY + I_o + \overline{X} - mY - M_o$$

$$(5)$$

General case

$$Y = c(Y) + I(Y) + \overline{X} - m(Y)$$

Equation 5 becomes upon solving for Y:

$$Y = \frac{C_o + I_o - M_o + \overline{X}}{1 + m - c - b} \qquad (6)$$

This result suggests that the equilibrium value of income equals the constant $(C_o + I_o - M_o + \overline{X})$ multiplied by

$$\frac{1}{1 + m - c - b} .$$

This last term is the multiplier of the system.

A multiplier in its broadest sense has no separate existence from a specific economic model to which it refers. Furthermore, within an economic model there are as many multipliers as possible marginal effects of a change of any economic variable upon another of which the first one is a component. Hence, the complexity or simplicity of a multiplier depends on the number of marginal relationships of economic variables under simultaneous consideration.

Naturally, distinguishing within any model between exogenous and endogenous variables one can investigate the effects of a change of an exogenous variable on one or more endogenous variables but not vice versa. A multiplier in that case can be only a derivative of an endogenous variable with respect to an exogenous one.

Let us consider the model summarized in identity $(\overline{S}')$. Differentiating the identity $(\overline{S}')$ with respect to Y, we obtain.

$$1 = \frac{dC}{dY} + \frac{dI}{dY} - \frac{dM}{dY} \qquad (6')$$

From Equation 6' we can easily derive expressions such as

$$\frac{dY}{dC} = \frac{1}{1 - I' + M'}\ ,\ \frac{dY}{dM} = \frac{1}{C' + I' - 1}\ ,$$

where

$$I' = \frac{dI}{dY}\ ,\ C' = \frac{dC}{dY}\ \text{etc.}$$

These expressions are the consumption multiplier, the import multiplier etc.

Using the method employed by C. Clark and Kahn, we write:

$$\frac{dY}{dI} = 1 + (C' - M') + (C' - M')^2 + (C' - M')^3 + \ldots.$$

$$\frac{dY}{dC} = 1 + (I' - M') + (C' - M')^2 + \ldots + \ldots$$

and so on. Now, on the basis of empirical evidence the normal case is for the term $(C' + I' - M')$ to be less than unity. For, the marginal propensity to consume is in the longer run certainly less than unity. The stability conditions require that $C' + I' < 1$, (or that b in the linear case be $1 - c$).

If we assume a positive marginal propensity to import the term $(C' + I' - M')$ is a <u>fortiori</u> less than unity.

Then, according to the rule of summation of the terms of a geometric progression any of the above series reduces to an expression such as:

$$\frac{dY}{dX} = \lim_{n \to \infty} \frac{1 - (C' + I' - M')^{n}}{1 - (C' + I' - M')} = \frac{1}{1 - C' - I' + M'} .$$

From this last expression we see that

$$dY = \frac{dX}{1 - C' - I' + M'} .$$

The meaning of this equation is clear. The increment in income-creating exports is multiplied by the factor

$$\frac{1}{1 - C' - I' + M'}$$

in order to give the equilibrium value of the income increase.

In the model presented, it was assumed that all exports are income-creating, all imports income-leakages, while no reference was made at all to sales or purchases on capital account which may be an important component of the foreign sector. It is proposed now to examine briefly this topic.

It is customary in the discussion of models of an open economy to include in exports and imports the items which make up the balance of trade - transactions on current account - without taking into account capital movements. This might be caused by the fact that the theory of capital movements is a complex one.

If one considers securities as commodities obeying the conventional rules of the items of the balance of trade, one should have little to modify in the general notation. An import of securities - a capital outflow should have a negative sign in the income-determination identity and an export of them should carry a positive sign.

But one has to keep in mind the following considerations.

The general notion that exports create income depends on an assumption about an economy's institutional behavior. It is, namely, assumed that the banking system of a country is always prepared to acquire foreign claims possessed by exporters as a result of their sales abroad against domestic currency which is going to be used for factor purchases in the production of exportables.

Imports, on the other hand, are considered leakages from the system because the banking system is supposed to absorb domestic currency in order to accommodate the importers demanding foreign values to pay for their imports. In other words, general convertibility of currencies is assumed. If one adopts the broad Keynesian definition of foreign lending as the sum of increments or decrements in balances held abroad, one sees that behind each current-account item movement there is always a corresponding capital movement either induced as in the case of a change in balances resulting from current transaction or autonomous. One now must state the conditions under which the opposite movements of various items in the balance of payments interact upon each other in affecting the net result.

This will enable one to refine the model by the introduction of parameters describing the role of exports and imports, defined broadly to include the movements of claims, in determining the level of income.

The distinction between induced and autonomous capital movements becomes necessary for the present purpose. Ragnar Nurkse (28) defines the induced capital movements as those "which result from changes in other items in the Balance of Payments". Carl Iversen (14) calls them "short-term equalizing capital movements".

The autonomous capital movements are defined as those resulting from a shift in the demand or supply functions for, or of, foreign balances and securities.

The theory of this sort of capital movement is a complex and controversial one and is not to be dealt with in this essay.

Carl Iversen (14) remarks: "...the long-term real capital movement..is temporarily offset by a short-term equalizing capital transfer in the opposite direction".

It is this autonomous kind of capital movements that is included in the income-determination identity as component of the exports and the imports.

This inclusion, however, is followed by some difficulties, especially as regards the import function. It was noted previously, that the relationship between imports and level of income is an established one in the analysis, but imports are defined in this context to refer only to the current account items. It might be assumed that with growing income there is a growing amount of foreign claims being accumulated indicating an increasing ability of the growing economy to lend. Besides, the realism of such an assumption is supported by the following consideration. The international flow of capital is a function of yield. As income increases, the supply of loanable funds increases leading to a decrease of the rate of interest which in turn induces capital movements out.

The question now is: If a spontaneous change occurs in the domestic demand for imports - including foreign assets - by which mechanism is the level of income affected and to which extent?

There is no unambiguous answer to this question. A taxonomic analysis is needed in order to consider plausible alternatives.

It all depends, then on whether the spontaneous* change in the import demand takes place at the expense of:

a. Idle funds

b. Bank debts

c. Domestic investment

d. Consumption

In the first two cases (idle funds and bank debts) the equilibrium level of income is not going to be affected, because aggregate demand is unaltered.

In the last two cases, or in any case in which domestic aggregate demand is partially reduced in order to finance the increase in autonomous imports, the equilibrium level of income is going to be negatively affected, assuming away for the time being, all possible effects from induced foreign

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*From the standpoint of history of economic thought it is to be noted that the autonomous movements of capital are not unanimously considered "autonomous" in the sense of being exogenous to a model of an open economy. An old and controversial subject is, indeed, whether the long-run capital movements adjust themselves to a given persistent imbalance in the other items of the balance of payments or whether it is the trade balance that follows the movements of capital, being really "autonomous" in the latter case. It is the classical view that the balance of trade is determined in the long run by independent capital movements - view shared by Taussig, Wicksell, Von Mises, Cassel, Angell, Ohlin, Iversen, Haberler, in the present century. J. M. Keynes "The German Transfer Problem" is the most distinguished representative of the non-autonomy of capital movements as defined in this footnote. The German literature on the topic in general adheres to this theory. For a discussion and a compromise position, see, J. Viner (35).

demand for exports.

The above instances can be most clearly analyzed by reference to the representation of macroeconomic equilibrium employing the Hicksian (12) LM and IS curves and broadening their meaning to take account of the foreign sector.

But before doing this, let us bring together the ingredients of the open economy to see what modifications are now needed. Use will be made of the linear case in order to achieve explicit solutions.

$$C = cY + C_o \tag{1a}$$

$$I = bY + I_o \tag{2}$$

$$M = mY + M_o \tag{3}$$

$$X = \overline{X} \tag{4}$$

$$Y = C + I + X - M \tag{5}$$

The modifications needed in order to employ the macroeconomic-equilibrium technique are due to the monetary sector. Here constant terms of trade are postulated in real terms in order to avoid introducing price-effects. Consequently, the import function will remain unchanged and only an additive term is to be included in the investment Equation 2. If i is the index-rate of interest, then

$$I = bY - hi + I_0 \tag{2}$$

where h is the slope of the investment function on a rate-of-interest - investment-spending surface at Y constant.

The money supply in the economy is assumed perfectly inelastic, given by

$$M^{(s)} = \overline{M}^{(s)}.$$

This amount is split into a component describing liquidity preference as a function of i, shaped in the traditional way Modigliani (25) and a component describing the amount desired for transaction purposes, as a function of real income,

$$M^{(T)} = M^{(T)}(Y)$$

so that

$$\overline{M}^{(s)} = M^{(a)}(i) + M^{(T)}(Y).$$

The equilibrium conditions in this macroeconomic model require both the real market and the money market to be in equilibrium. At any level of Y the real market is in equilibrium when the supply is equal to the demand made up by domestic investment spending, consumption spending and spending on the production of exportables. The money market is in equilibrium when

$$M^{(s)} = M^{(d)},$$

when the supply of money equals the demand for it. The loci of points at which income is in equilibrium with reference to the real market and money market are the IS and LM curves, respectively.

The equation of the IS curve is:

$$Y = \frac{C_o + I_o - M_o + \overline{X}}{1 + m - c - b} - \frac{hi}{1 + m - c - b}$$

The equation of the LM curve is given by the following expression when

$$M^{(T)} = v_1 Y$$

and

$$M^{(a)} = v_o - v_2 i:$$

$$Y = \frac{\overline{M}^{(s)} - v_o}{v_1} + \frac{v_2}{v_1}.i$$

The simultaneous solution of these two equations yields the equilibrium level of income, Ye.

Now, let us consider again the autonomous increase in the demand schedule for imports financed by a. idle funds, b. by bank debts, c. domestic reduction on investment spending, d. reduction on domestic consumption.

a. In this case the position of IS curve will remain unaffected, since the real market does not get disturbed. Most likely no movement will be noticed along the LM curve, either.

b. Neither of the components of $\overline{M}^{(s)}$ is affected, nor IS. Ye remains the equilibrium level.

c. and d. In these cases, the IS curve moves to the left (I'S') and the equilibrium level of income is negatively affected.

If a spontaneous increase occurs in the demand for exports, assuming perfectly elastic supply of resources, effective demand will rise in the first round by the amount of the increase.

So far, however, as an increase in exports is defined to include exports of goods and services plus securities, not all of exports may be expected to be income-creating. Let us define k the fraction of exports that is income-creating. Similarly, let σ mean the fraction of imports that represent a leakage from the income stream.

The income-determination identity, after taking into account the two new parameters, k and σ, becomes;

$$Y = C + I + kX - \sigma M \tag{7}$$

and the equilibrium level of income is given by;

$$Y = \frac{1}{1 - c - b + \sigma m} \cdot (C_0 + I_0 - \sigma M_0 + kX). \quad (8)$$

On the basis of this last equation, we can determine the effects on income of changes in the relevant parameters and variables.

A change in autonomous imports will affect the equilibrium level of income, as given by:

$$\Delta Y = \frac{-\sigma}{1 - c - b + \sigma m} \Delta M_0 \quad (9)$$

We assume that the multiplier is positive. The change in income, Equation 9, then is negative if ΔM_0 is positive and vice versa.

A change in the slope of the import function, Δm, affects income as indicated by:

$$\Delta Y = \frac{-\sigma}{1 - c - b + \sigma m} \Delta mY \quad (10)$$

If Δm is positive, income declines, if Δm is negative income increases.

It is then likely for σ to be $0 < \sigma < 1$. For its value to be equal to unity, one ought to assume a change in imports resulting from a change in the relative prices of identical or close substitute commodities produced both at home and abroad. σ would equal zero if the change in the demand for imported commodities is financed entirely at the expense of idle funds and bank debts.

A change in the parameter σ, indicating a change in the degree of substitution between imports and domestic expenditure, is described by the equation:

$$\Delta Y = \frac{-1}{1 - c - b + \sigma m} \cdot \Delta\sigma (M_0 + mY) \qquad (11)$$

If $\Delta\sigma$ is negative income increases, if it is positive income decreases.

A change in exports results in:

$$\Delta Y = \frac{1}{1 - c - b + \sigma m} \; k\Delta X \qquad (12)$$

which has the sign of ΔX.

If the fraction of income-creating exports over the total of exports changes, then

$$\Delta Y = \frac{1}{1 - c - b + \sigma m} \Delta kX \qquad (13)$$

which has the sign of Δk. The above expressions, Equations 9 through 13 are, naturally conditioned by a _ceteris paribus_ assumption.

The effects on income of simultaneous equal changes in exports and imports were not clearly analyzed, until fairly recently.* Thus Stolper (34) writes:

> "...A simultaneous increase in imports and exports will have an expansionary effect if it is not offset by downward changes in the average propensity to consume domestic goods... The best way to describe the effects of trade balances is by means of the marginal propensity to import. The best way of describing the effect of the volume of trade as distinct from the trade balances is by means of the average propensity to spend. For a discussion of the full effects of foreign trade on national income both average and marginal propensities have to be considered".

Let us now consider the effect of equal changes in exports and imports on Y. $\Delta X = \Delta M$ ex hypothesi; ΔY then is given by:

$$\Delta Y = \frac{k - \sigma}{1 - c - b + \sigma m} \cdot (\Delta X) \tag{14}$$

Equation 13 easily leads to the following taxonomic analysis. First let us assume, for simplicity $k = 1$, so that

$$Y = \frac{1 - \sigma}{1 - c - b + \sigma m} \quad \Delta X. \tag{14'}$$

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*Haberler argued in his Prosperity and Depression that only an increase in exports had expansionary effects on the national income. The subsequent discussion, Stolper (34), Polak (29), clarified many obscure points. It seems however that the rigorous and simple methodological approach to the problem became possible after the publication of Haavelmo's study (9) on fiscal policy.

If the balanced change is a decline in trade, the above equation becomes

$$\Delta Y = \frac{\sigma - 1}{1 - c - b + \sigma m} \Delta X. \tag{15}$$

If the change represents an increase in trade, Equation 13 remains unchanged.

In case of Equation 14 if the substitution between domestic and foreign commodities is indicated by $\sigma > 1$, income will increase, while it will decrease in case of Equation 13.

In other words, if the degree of substitution between domestic and foreign commodities is greater than unity a decline in foreign trade will lead to a rise in income and an expansion in foreign trade will lead to a decrease in income.

If the elasticity of substitution is unity, the volume of trade, in case of balanced changes will have no effect on income, so that $\Delta Y = 0$, the multiplicand being zero.

If the degree of substitution is less than unity, which may be considered to be the normal case, an expansion in balanced foreign trade will lead to an increase in income and a decline in it will result in contraction of Y, in the first case by being positive Equation 13 in the second Y being negative Equation 14.

Regarding the effect of simultaneous changes in exports and imports of different relative magnitudes it can be stated in general, on the basis of the Equation 16 below, that:

$$\Delta Y = \frac{k \Delta X - \sigma \Delta M}{1 - c - b + \sigma m} . \tag{16}$$

If the change in the volume of trade is positive, income increases if

$$\frac{k}{\sigma} > \frac{\Delta M}{\Delta X} ,$$

decreases if

$$\frac{k}{\sigma} < \frac{\Delta M}{\Delta X}$$

and remains unaffected if

$$\frac{k}{\sigma} = \frac{\Delta M}{\Delta X} .$$

If the change in the volume of foreign trade is negative income will increase, if

$$\frac{k}{\sigma} < \frac{\Delta M}{\Delta X} ,$$

decrease if

$$\frac{k}{\sigma} > \frac{\Delta M}{\Delta X}$$

and remain unchanged if

$$\frac{k}{\sigma} = \frac{\Delta M}{\Delta X} .$$

Dynamic approach

Up to this point we were not concerned with timing the variables of the simple model of an open economy. It is, however, evident that the relationships described above must involve time in an essential way in any attempt to explain more closely the behavior of an economy. This makes for a dynamic or, to use Tintner's more modest term, a non-static model.

The effects of lags in the variables of a model have been investigated in both theoretical and econometric studies and their policy implications indicated.

Whatever the kind of lags one may introduce, one can state in general, that the equilibrium value of income, if such a value exists, is determined by the coefficients of the behavioral functions and is not affected by the lags.

The latter give rise to various types of paths towards equilibrium - if again the coefficients of the equations permit an equilibrium - generating damped cycles or conditioning a smooth asymptotic approach to it.

The independence of the equilibrium from lags can be very easily seen in the following manner. Let it be supposed that a set of n equations describing an economy, is finally incorporated by substitution in the income-determination equation, being a non-homogenous difference equation of n^{th} degree

$$y_t = c_1 y_{t-1} + c_2 y_{t-2} + \dots + c_n y_{t-n} + A_t.$$

Let it be now assumed that equilibrium exists, being equal to $\bar{y}$. Then this value can be expressed by substituting it for the variables in the above equation and solving explicitly for it:

$$\bar{y} = \frac{A}{1 - c_1 + c_2 - \dots - c_n}.$$

The multiplier of the system exists if the expression

$$(1 - c_1 - c_2 \dots - c_n) > 0.$$

The multiplier, then as a finite parameter, depends on the determinateness of the equilibrium of the system. It follows therefrom that lags are not needed in order to assess the ultimate effect on income of a change in the variables concerned. Nevertheless, it is very interesting to see how the behavior of the model is affected by its foreign sector.

Let us start by restating the income-determination equation with its components dated

$$Y_t = C_t + I_t + X_t - M_t. \tag{1b}$$

The Robertsonian assumption will be made that present consumption is a linear function of the previous-period income.

$$C_t = cY_{t-1} \tag{2b}$$

The treatment of investment gives ground for many classifications. So, current investment can be expressed as a function of the previous-period level of income or by an acceleration equation. In the latter case, the order of lags may serve as the basis of further distinction, namely, between models of the Harrod-Domar type and Samuelson-Hansen-Hicks-Metzler type.

The Harrod-Domar type of investment function admits of one lag

$$I_t = b\ (Y_t - Y_{t-1}) \tag{3b}$$

where b is the acceleration coefficient.

It is to be noted that Equation 3b is called a Harrod-Domar type investment function only on the basis of the order of lags. If the hypothesis is that current investment is induced by changes in income, this does not reflect the spirit of Domars contribution. The correct interpretation of Equation 3b according to the model produced by Domar would be: I_t is the exogenous variable; current investment I_t increases capacity by the ratio

$$I_t \frac{1}{b}$$

which, if fully utilized, equals ΔY or

$$Y_t - Y_{t-1}.$$

Then,

$$I_t = b\Delta Y,$$

is a relationship in which the direction of causation is: given investment, the potential change in income is determined.

The import function may assume the form of the consumption function

$$M = mY_{t-1}. \tag{4b}$$

Substituting the above equations into Equation 1b, we get

$$Y_t = cY_{t-1} + b\ (Y_t - Y_{t-1}) + X_t - mY_{t-1} \tag{5b}$$

and

$$Y_t = \frac{c - b - m}{1 - b} Y_{t-1} + X_t. \tag{6b}$$

Equation 6b is a first-order, linear, non-homogeneous difference equation. The solution to its homogeneous part is:

$$Y_t^{(H)} = Y_o \left(\frac{c - b - m}{1 - b} \right)^t \tag{7b}$$

where Y_o is the initial condition. It is the homogeneous part that affects the stability of the system.

On the basis of Equation 7b, we observe that there are several possible behavior patterns of the system.

If the expression in parenthesis is greater than unity, income grows exponentially over time.

If it is equal to unity, the contribution of the homogeneous part is Y_o and income will be ultimately determined by the additive non-homogeneous component X_t.

If the expression in parenthesis is between zero and unity, the initial condition loses in importance over time and income approaches some multiple value of the exports component X_t.

$$Y_t \to X_t \left[\frac{1 - \left(\frac{c - b - m}{1 - b} \right)^t}{1 - \left(\frac{c - b - m}{1 - b} \right)} \right] \xrightarrow[\text{dst} \to \infty]{} X_t \left[\frac{1}{1 - \frac{c - b - m}{1 - b}} \right].$$

If the expression in question is negative, oscillations are bound to occur, which will be either explosive, if its absolute value is greater than unity, or damping for absolute values less than unity.

One can now concentrate on the role of the parameter m, the marginal propensity to import, as to the possible behavior patterns of the model.

If income is to grow exponentially, m must satisfy the condition

$$m < c - 1.$$

In order for income to depend with respect to its ultimate equilibrium value on its autonomous component X_t, given Y_0, the condition

$$m = c - 1,$$

should be satisfied.

If income is to depend ultimately on its autonomous part X_t, the influence of the initial condition vanishing over time, the following two conditions must be fulfilled simultaneously:

$$m > c - 1$$

$$m < c - b$$

If the system explodes in oscillations,

$$m > 1 - c + 2b.$$

The model does not include any built-in stabilizers. In order to preserve economic sense, one may disregard the cases of explosive oscillations and exponential growth, both being perfectly unrealistic.

For an economy to be perfectly rigid, without multiplier effects, the marginal propensity to import is seen to be negative and equal to minus the propensity to save, if as normally assumed the marginal propensity to consume is less than unity and greater than zero.

If an economy is less rigid, allowing for multiplier effects, the marginal propensity to import must be larger than minus the marginal propensity to save but less than the difference between the marginal propensity to consume and the acceleration coefficient.

Although these results are not striking at all as intuitively obvious, it can be said that they correspond to a "reasonable" type of economy. For such a type may be conceived as tending towards some stagnation level of income, unless autonomous injections pull it ahead. This is the case of the conditions discussed last. The values of m as limited by the conditions implying such an economy are acceptable, if one considers the results of some empirical work done in this field, as will be seen later. So, if one assumes a marginal propensity to consume of .85 and an acceleration coefficient of .80, within the framework under discussion, the marginal propensity to import must be larger than -.15 and less than .5. Given the marginal propensity to consume, the higher is the coefficient describing the response of investors to income changes, the smaller the marginal propensity to import has to be in order to meet the requirements of such an economy, and vice versa. The higher however is the marginal propensity to consume, given an acceleration coefficient, the larger are the possible values of m compatible with the behavior pattern assumed.

It is intended now to consider a model of an open economy in which investment is a function of the change in income from period t-2 to period t-1.

As indicated earlier, there is a number of versions of this type of models referring to a closed economy.

The Samuelson-Hansen type, Samuelson (31), follows the Austrian tradition in assuming that investment provides for consumption goods only.

The investment function is given in terms of the change in consumption at the relevant time periods and through the lagged consumption function of the Robertsonian type one can obtain an expression in terms of income.

Metzler (20) and Hicks (13) work with an investment function directly in terms of income changes.

In this model we follow Metzler and Hicks.

The income identity is:

$$Y_t = C_t + I_t + X_t - M_t. \tag{1c}$$

The consumption function is again of the Robertsonian type,

$$C_t = cY_{t-1}. \tag{2c}$$

Before writing the investment function, one must make explicit the assumptions:

a. Labor is redundant, in the Harrod-Domar fashion.

b. Capital is used in fixed proportions.

c. There is some optimal capital-output ratio.

The investment function is,

$$I_t = b(Y_{t-1} - Y_{t-2}). \tag{3c}$$

The import function assumes the same form again,

$$M_t = mY_{t-1}. \tag{4c}$$

Substituting into Equation 1c, it obtains

$$Y_t = (c + b - m)\, Y_{t-1} - bY_{t-2} + X_t. \tag{5c}$$

Assuming that equilibrium exists and is Y^*,

$$Y_t = Y^* = \frac{1}{1 - c + m} X_t,$$

which represents the familiar multiplier expression. It is to be noted that the acceleration coefficient does not appear in the equilibrium expression and consequently it does not contribute to its value.

Again, the non-homogeneous part of Equation 5c X_t can be proven not to influence the behavior of the system.

It is the homogeneous part of Equation 5c that determines the time path of income depending on the roots of its characteristic equation.

Let a solution be assumed to be:

$$Y_t^{(H)} = (c + b - m)\, Y_{t-1} - bY_{t-2},$$

$$Y_t^{(H)} = z^t.$$

The characteristic equation then is:

$$z^2 - (c + b - m)\, z + b = 0 \qquad (6c)$$

The value of z is given by the formula

$$z = \frac{c + b - m \pm \sqrt{(c + b - m)^2 - 4b}}{2} \lesseqgtr 1. \qquad (7c)$$

One observes that the roots will have an imaginary part if the discriminant is less than zero.

If the discriminant is zero the one root is real and gives rise to behavior patterns as determined by:

$$Y_t = (A_t^n + B)\, Z_o^t. \qquad (n = 1,2,\ldots)$$

where A and B are constants determined on the basis of initial conditions, and Z_o is the value of the root.

The first problem then is to concentrate on whether

$$\frac{c + b - m}{2} \lesseqgtr 1.$$

The system explodes if the above expression is greater than unity. For this to happen,

$$c + b - 2 > m.$$

In this unrealistic case the values of m are restricted most likely to negative ones.

If $m = c + b - 2$, when the root is unity, the system progresses or declines linearly if $n = 1$ depending on whether the initial conditions determine a positive or a negative value for A, or exponentially for $n > 1$.

If the root is less than unity,

$$m > c + b - 2.$$

For given positive values of the marginal propensity to consume and the acceleration coefficient, it is evident that within a certain range the larger the marginal propensity to import the greater is the stabilizing influence.

In order for explosive oscillations to be avoided the condition below has to be fulfilled no matter how large n is:

$$c + b > m > c + b - 2$$

The values of the marginal propensity to import as restrained by the last condition for stability are given quite a comfortable range.

Now suppose that the discriminant is negative. In this case the solution to the homogeneous part of the income determination equation will be a pair of complex conjugate numbers.

Let

$$\frac{\sqrt{(c + b - m)^2 - 4b}}{2} \equiv iD \text{ and } c + b \equiv E.$$

Then,

$$Z = \frac{E - m}{2} \pm iD$$

$$Z_1 = \frac{E - m}{2} + iD$$

$$Z_2 = \frac{E - m}{2} - iD$$

and

$$Y_t^{(H)} = A(Z_1)^t + B(Z_2)^t.$$

The last expression becomes:

$$Y_t^{(H)} = A\left[\sqrt{\left(\frac{E - m}{2}\right)^2 + D^2}\right]^t (\cos t\theta + i\sin t\theta) +$$

$$B\left[\sqrt{\left(\frac{E - m}{2}\right)^2 + D^2}\right]^t (\cos t\theta - i\sin t\theta) .$$

Write

$$\sqrt{\left(\frac{E - m}{2}\right)^2 + D^2} \equiv R, \quad \text{the modulus:}$$

Then,

$$Y_t^{(H)} = R^t \left[(A + B) \cos t\theta + i (A - B) \sin t\theta\right]$$

which is the final solution.

If the discriminant is positive,

$$Y_t^{(H)} = A(Z_1^*)^t + B(Z_1^*)^t$$

where Z_1^* and Z_2^* are the two roots. In each of the above cases A and B are arbitrary constants determined by the initial conditions. The non-homogeneous part will at most be of the form

$$Y_t^{(P)} = Kt^n \qquad (n = 0,1\ldots)$$

where n is not greater than the order of the equation under consideration. The arbitrary constants A and B are assigned specified values after a particular solution has been found.

The interpretation of the solutions is as follows: Consider first the complex roots case.

The factor

$$\left(A + B\right)\cos t\theta + i\left(A - B\right)\sin t\theta$$

repeats itself oscillatorily. By itself, it neither explodes nor dies out. Its presence in the solution of a system indicates that fluctuations will occur. The nature of the oscillations in terms of trend will depend on the factor R^t. If $R = 1$, it will have no influence at all on the behavior of the model and the periodic movement as conditioned by the trigonometric term will perpetuate itself.

If

$$R > 1, \quad R^t \longrightarrow \infty \text{ as } t \longrightarrow \infty .$$

The situation is explosive the fluctuations becoming intensified.

If

$$R < 1,$$

the fluctuations become damped over time converging to an equilibrium path.

It is the latter case which appears more realistic. The model may exhibit this kind of stability as it will be seen in examining the components of the modulus R.

$$R = \sqrt{\left(\frac{E - m}{2}\right)^2 + \frac{(E - m)^2 - 4b}{4}} = \sqrt{\frac{(E - m)^2 - 2b}{2}}$$

It is very likely that the numerator inside the radical is greater than zero and less than unity in most cases except possibly for very short periods.

For a positive modulus, given an aggregate marginal propensity to expend, the larger the marginal propensity to import, the shorter the time period needed for the income path to converge to the equilibrium path.

The particular solution adds either a constant term or a trend term. No essential modifications in the conclusions are needed.

If the discriminant is positive, and both roots

$$0 < z_1^*, z_2^* < 1,$$

then it is the particular solution which determines the equilibrium solution. Negative roots give rise to oscillations which are explosive if the dominant root has an absolute value greater than unity. If the dominant root is unity a constant becomes ultimately the solution consisting of the particular solution and the coefficient of the dominant root.

In general, there is a c - b - m -space within which any combination of c, b, m-values corresponding to any point in a subspace bounded by the surface

$$(c + b - m)^2 - 4b = 0$$

gives rise to oscillatory movements, either tending and reaching eventually an equilibrium value depending on whether the dominant root has absolute value less than unity, or exploding when the dominant root is greater than 1. Nonoscillatory behavior arises from combinations of the parameter values above the mentioned surface, but again stability requires the dominant root to be less than unity.

Upon observing the formula of the second-degree polynomial giving the roots of the characteristic equation of the system, one sees that the parameter m assumed to be positive, is a stabilizer of the model reducing the absolute value of the roots for realistic values of c and b.

The above conclusions about the role of the marginal propensity to import in dynamic systems constructed on the basis of the acceleration principle need no essential modification if we are to consider additional hypotheses such as the ones leading to a flexible accelerator Ferguson (7) or a variable one.

A TWO-COUNTRY INTERNATIONAL SYSTEM

In what preceded a static model of an open economy with sectors, consumption, investment and foreign trade was set up.

The income-creating mechanism of exports was examined in some detail defined broadly to include exports of domestic assets or imports of capital and the income-leaking effects of imports defined broadly to include imports of foreign assets or exports of capital. After a static-macroeconomic-equilibrium situation was presented the dynamic version of this model was considered with view on the effects of the marginal propensity to import on macroeconomic stability, <u>ceteris paribus</u>.

Now, it is proposed to set up a simple model of a two-country international economic system and examine the effects on income of some autonomous shocks.

Up to this point the export component of income was a datum. Now that an international economic system is considered, exports will be incorporated into the model as an endogenous element.

The autonomous shocks that may be examined fall broadly into three basic categories:

a. There is an internal change to be considered occurring in the level of a country's effective demand as a function of that country's income.

b. At a given level of income, autonomous disturbances may take place in the form of shifts in the distribution of expenditures on goods and services produced by one of the two countries to the goods and services produced by the other.

c. Changes in the movements of capital indicating increases or decreases in foreign lending or changes in unilateral transactions such as aid or capital transfers etc.

The setting of the inquiry is characterized again by a substantial amount of unemployment of resources and consequently elastic supply of them within a wide range; by money-wage inflexibility and by sufficient reserves in order to rule out changes in the exchange rates. The aggregate production function of the economies is assumed to be homogeneous of the first degree.

A consumption function will be postulated in its general form, an investment function with argument the level of income, and the expressions relating to the foreign sector of each country. In the notation number subscripts are affixed to variables referring to the second country.

So, $C = C(Y)$ is the consumption function of the first country, $C_1(Y_1)$ the consumption function of the other, $I(Y)$ and $I_1(Y_1)$ the investment functions, respectively, relating the amount of investment to the level of income of each country, $M(Y)$ is the import function of the first country and export function of the other while $M_1(Y_1)$ is the export function of the first country and the import function of the second country. The above equations determine the equilibrium position of the system, provided that the stability conditions are satisfied.

The equilibrium of any static economic system is connected with the assumptions underlying the corresponding dynamic system; namely it is not possible to know the nature of this equilibrium unless postulates are specified about the dynamic behavior of the system.

This is the so-called correspondence principle.

In general, the formulation of an economic static system is followed by a comparative statics question. In other words, the values of the variables determined by the static system are differentiated with respect to the parameters. The sign of the derivatives will indicate the direction of the change in the variables as a consequence of a small displacement in the parameters. In many cases, however, the sign cannot be determined unambiguously, because of missing information about the functional relationships entering into the system.

The correspondence principle can then be used in order to provide this needed information.

The dynamic system can be derived in the case of this model if the following assumptions are made:

1. The consumption of goods and services lags one period behind income - and hence the imports for consumption purposes, which are the only kind of imports assumed to take place.

2. Entrepreneurial decisions with respect to investment are based on expectations formed on the ground of information about the level of income of the previous period.

The dynamic model then is:

$$C_t = C(Y_{t-1}) \tag{1}$$

$$I_t = I(Y_{t-1}) \tag{2}$$

$$M_t = M(Y_{t-1}) \tag{3}$$

$$C_{1t} = C_1(Y_{1t-1}) \tag{4}$$

$$I_{1t} = I_1(Y_{1t-1}) \tag{5}$$

$$M_{1t} = M_1(Y_{1t-1}) \tag{6}$$

From the definition of income, the above equations are consolidated into:

$$Y_t = C(Y_{t-1}) + I(Y_{t-1}) + M_1(Y_{1t-1}) - M(Y_{t-1}) \tag{7}$$

$$Y_{1t} = C_1(Y_{1t-1}) + I_1(Y_{1t-1}) + M_1(Y_{2t-1}) + M(Y_{t-1}) \quad (8)$$

The explicit solutions to Equations 7 and 8 will enable one to state the conditions which the functional relationships must fulfill for the international economic system to be stable.

Expanding Equations 7 and 8 in a Taylor series and dropping all non-linear terms it obtains:

$$\begin{aligned} Y_t - Y_o = {} & C(Y_o)' \ (Y_{t-1} - Y_o) + I(Y_o)' \ (Y_{t-1} - Y_o) \\ & + M_1(Y_{1o})' \ (Y_{1t-1} - Y_{1o}) - M(Y_o)' \ (Y_{t-1} - Y_o) \end{aligned} \quad (9)$$

and

$$\begin{aligned} Y_{1t} - Y_{1o} = {} & C_1(Y_{1o})' \ (Y_{1t-1} - Y_{1o}) \\ & + I_1(Y_{1o})' \ (Y_{1t-1} - Y_{1o}) + M(Y_o)' \ (Y_{t-1} - Y_o) \\ & - M_1(Y_{1o})' \ (Y_{1t-1} - Y_{1o}), \end{aligned} \quad (10)$$

where $C(Y_o)'$ indicates first differentiation and evaluation at Y_o, and similarly for the other terms.

Equations 9 and 10 are difference equations whose solutions can be obtained in the usual way considering the derivatives of the expressions as constant coefficients.

In case no trade existed between the two countries the income path in each country is given by the equations

$$Y_t = Y_o + C(Y_o)' + I(Y_o)' \; (Y_{t-1} - Y_o)$$

$$Y_{1t} = Y_{1o} + C_1(Y_{1o})' + I_1(Y_o)' \; (Y_{1t-1} - Y_{1o}).$$

The solution to this equation is of the form:

$$Y_t = Y_o^{*} + \left(Y(0) - Y_o\right)(C' + I')^t$$

where I' and C' are

$$\frac{dI_t}{dY_{t-1}}$$

and

$$\frac{dC_t}{dY_{t-1}},$$

respectively.

In order that the income path converges to some equilibrium value, $(C' + I')$ must be less than unity in absolute value which is the usual stability requirement, encountered throughout.

If country Y trades with the rest of the world in general in a way that its exports may be considered a datum, as assumed in the first section of this essay, the linear difference equation obeys the same conditions for stability, the root now being, $(C' + I' - M')$, where

$$M' = \frac{dM_t}{dY_{t-1}} .$$

In an economy then where imports are a positive function of income a large marginal propensity to import is likely to counterbalance the destabilizing effects of excessive aggregate propensities to spend, or more correctly the higher is the foreign component in the aggregate propensities to spend, the stabler _ceteris paribus_ the economy is bound to be.

In the general case however, where possible reactions of the rest of the world may be regarded essential, or in the case, more specifically, of a two-country model where the flows of goods and services into either country are to a high degree interdependent, the time path of incomes, Y and Y_1, cannot be determined without taking into account the nature of the reactions in question. In formal terms, since again the dynamic behavior of the model is determined by the homogeneous paths of the equations, for small shocks or deviations from the equilibrium - on the assump- it exists - the time path of Y and Y_1 will be conditioned by the following expressions:

$$Y_t^{(H)} = A p_1^{\,t} + B p_2^{\,t} \tag{11}$$

$$Y_{1t}^{(H)} = C p_1^{\,t} + D p_2^{\,t} \tag{12}$$

where the p's are the roots of the characteristic equation of the system.

The latter is given by the quadratic equation derived from Equations 9 and 10. In determinantal form:

$$\begin{bmatrix} (C' + I' - M')-p & M_1' \\ M' & (C_1' + I_1' - M_1')-p \end{bmatrix} = 0 \tag{13}$$

For stability, p's < 1.

The necessary and sufficient conditions for this can be stated as follows:

$$(C' + I' - M') + (C_1' + I_1' - M_1') < 2 \tag{i}$$

which states that if a system is stable the trace of the matrix of its coefficients - which is equal to the absolute value of the sum of the roots - must be less than the degree of the system, and

$$(C' + I' - M')\,(C_1' + I_1' - M_1') - M_1'M < 1 \tag{ii}$$

which means that the characteristic determinant of the coefficients of the system - being equal to the product of the roots - must be less than unity.

The economic implications of the above formal conditions for stability can be readily derived. So, it is apparent that at least one country, must be stable, for the international economic system to be stable. If both countries are stable, this implies that the system as a whole is stable.

Roughly speaking, the instability of the one country can be absorbed by the stability of the other, provided that the latter's low propensities are low enough for the former's excessive ones.

For instance, let it be assumed that

$$C' + I' - M' = 1.05,$$

$$M' = .06$$

and

$$C'_1 + I'_1 - M'_1 = .95,$$

$$M'_1 = .1.$$

For condition ii to be satisfied

$$(1.05)\ (.95)\ -\ (.06)\ (.1) < 1$$

Indeed, the propensities as assumed above imply a stable international system since

$$0.9915 < 1.$$

Clearly, condition ii implies that given an excessive aggregate propensity to expend on domestically produced goods and services of the one country, the aggregate propensity to expend on domestically produced goods and services of the other country must be low, for international stability. The more excessive the magnitude of the former, the lower the magnitude of the latter must be. This is what is meant by the idea that the instability of the one country can be absorbed by the stability of the other provided the relevant propensity of the latter is low enough.

Here, the following point deserves mention. It must be remembered that macroeconomic instability of one country in the sense used in this essay results from an excess over unity of the aggregate propensity to expend on domestically produced goods and services. When this is the case, equilibrium is unattainable within an economically acceptable range, although it exists mathematically, since the multiplier is a finite negative quantity. It follows, that it does not make sense to discuss the case of a country under these conditions and in the example used above the aggregate propensity 1.05 seems to lead to absurd results. This however is not so, because macroeconomic instability of a country in isolation does not necessarily imply macroeconomic instability of an international system where the variables of each country are determined simultaneously, a situation which is stated by condition ii above.

In the real world, when one takes account of more variables which enter into the picture, the equilibrium of the system is assured by the movement of relative prices, and opposite international movements of capital, as well. In the above model, where only income effects are considered, still equilibrium is possible under mathematically stated conditions through the repercussions on, and of, the foreign sectors of the countries in question. Excessive propensities to expend on domestic commodities in one country lead to increases in income of the same country, on which the imports depend positively. Increased imports represent an increased leakage out of the circular flow of the economy in consideration and an injection into the circular flow of the economy of the other country. This interplay through the foreign sectors of the two economies makes it possible for international equilibrium to be established, under the conditions explicitly stated above.

With the above information, one can determine the features of the static system. The values of the variables at the equilibrium position will however not satisfy the values required for a new equilibrium, if this new equilibrium is possible, after shifts occur in the parameters of the system.

It will now be examined how autonomous shocks affect the economies of the countries of the international economic system.

It has already been indicated what types of shocks are going to be considered. Let first changes in the level of effective demand be taken into account. Such changes may be due to several general factors. An innovation, for instance, may affect the aggregate cost of a certain type of investment moving the marginal-efficiency-of-capital schedule to the right by a distance a. Or any component of the cost of raising funds for investment, may be reduced, say, by an autonomous change in credit policy, followed by the same effect of shifting the marginal-efficiency-of-capital schedule positively.

Let it be supposed that such a shift occurs in country Y. Its investment function can be then written

$$I = I(Y) + a$$

and the system is:

$$Y = C(Y) + I(Y) + M'(Y') - M(Y) + a \qquad (14)$$

$$Y_1 = C_1(Y_1) + I_1(Y_1) + M(Y) - M_1(Y_1) \qquad (15)$$

By differentiating Equations 14 and 15 with respect to a, it will be possible to trace the effect of the autonomous shock throughout the international economy.

$$\frac{\partial Y}{\partial a} = \frac{\partial C}{\partial Y}\frac{\partial Y}{\partial a} + \frac{\partial I}{\partial Y}\frac{\partial Y}{\partial a} + \frac{\partial M_1}{\partial Y_1}\frac{\partial Y_1}{\partial a} - \frac{\partial M}{\partial Y}\frac{\partial Y}{\partial a} + 1 \qquad (16)$$

$$\frac{\partial Y_1}{\partial a} = \frac{\partial C_1}{\partial Y_1}\frac{\partial Y_1}{\partial a} + \frac{\partial I_1}{\partial Y_1}\frac{\partial Y_1}{\partial a} + \frac{\partial M}{\partial Y}\frac{\partial Y}{\partial a} - \frac{\partial M_1}{\partial Y_1}\frac{\partial Y_1}{\partial a} . \qquad (17)$$

The unknowns of the above system are

$$\frac{\partial Y}{\partial a}$$

and

$$\frac{\partial Y_1}{\partial a} ,$$

which measure the effect of a shift in the marginal-efficiency-of-capital schedule of the first country on the two countries' incomes. The other partial derivatives are the parameters of the static system which are assumed to be known.

Upon, rearranging Equations 16 and 17, it obtains:

$$(1 - C' - I' + M')\frac{\partial Y}{\partial a} - M_1'\frac{\partial Y_1}{\partial a} - 1 = 0 \qquad (16')$$

$$(1 - C_1' - I_1' + M_1')\frac{\partial Y_1}{\partial a} - M_1\frac{\partial Y}{\partial a} = 0. \qquad (17')$$

In matrix notation, Equation 16' and 17' become:

$$\begin{bmatrix} 1 - C' - I' + M' & - M_1' \\ - M' & 1 - C_1' - I_1' + M_1' \end{bmatrix} \begin{bmatrix} Y \\ Y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad (18)$$

where

$$C' = \frac{\partial C}{\partial Y}$$

etc.,

$$C_1' = \frac{\partial C_1}{\partial Y_1}$$

etc.,

$$Y_a = \frac{\partial Y}{\partial a}$$

and

$$Y_{1a} = \frac{\partial Y_1}{\partial a}.$$

Solving for the unknowns by Cramer's Rule, one gets:

$$Y_a = \frac{\begin{vmatrix} 1 & -M_1' \\ 0 & 1 - C_1' - I_1' + M_1' \end{vmatrix}}{D} = \frac{1 - C_1' - I_1' + M_1'}{D} \qquad (19)$$

and

$$Y_{1a} = \frac{M'}{D} \qquad (20)$$

where D is the determinant of the matrix of the system of coefficients

$$\begin{vmatrix} 1 - C' - I' + M' & -M_1' \\ -M' & 1 - C_1' - I_1' + M_1' \end{vmatrix}$$

Two more sub-systems will now be examined referring to the sectors of each economy and the effects of the shift in the marginal-efficiency-of-capital schedule in question. The first sub-system is:

$$C^* = C(Y) - M(Y) \qquad (21)$$

$$I^* = I(Y) + M_1(Y_1) + a \qquad (22)$$

Where C^* is defined to be the difference between aggregate consumption and consumption of imported goods and services.

$$C_a^* = C'Y_a - M'Y_a = Y_a\ (C' - M') \tag{21'}$$

$$I_a^* = I'Y_a + M_1'Y_{1a} + 1 \tag{22'}$$

Where

$$C_a^* = \frac{\partial C^*}{\partial a}$$

and

$$I_a^* = \frac{\partial I^*}{\partial a},$$

the effects namely of the shift on the difference between aggregate consumption and consumption of foreign commodities and on investment expenditures, respectively.

Substituting into Equation 21' the value of Y_a from Equation 19, we have

$$C_a^* = (C' - M')\ \frac{1 - C_1' - I_1' + M_1'}{D} \tag{23}$$

Substituting into Equation 22' from Equations 19 and 20 for the values of Y_a and Y_a' respectively, we obtain:

$$I^* = I'\ \frac{1 - C_1' - I_1' + M_1'}{D} + M'\ \frac{M'}{D} + 1 \tag{24}$$

Similarly, the sub-system of the sectors of the other country is:

$$C_1^* = C_1(Y_1) - M_1(Y_1) \tag{25}$$

$$I_1^* = I_1(Y_1) + M(Y) \tag{26}$$

Going exactly through the steps of the immediately previous case, the following results are derived:

$$C_{1a}^* = C_1' Y_{1a} - M_1' Y_a = Y_{1a} \; (C_1' - M_1') \tag{25'}$$

$$I_{1a}^* = I_1' Y_{1a} + M' Y_a \tag{26'}$$

and

$$C_{1a}^* = \frac{M'}{D} \; (C_1' - M_1') \tag{27}$$

$$I_{1a}^* = I_1' \; \frac{M'}{D} + M_1' \frac{1 - C_1' - I' + M'}{D} = M' \; \frac{1 - C_1' + M_1'}{D} \; . \tag{28}$$

Formal expressions are thus obtained about the effects of an autonomous shock of the type described on the two economies Y and Y_1 and the sectors which were taken to constitute them.

In the attempt to evaluate these effects, one meets the difficulty of not knowing the sign of the determinant D.

One can however get out of this difficulty by assuming stability of the two economies when no foreign trade takes place, so that $(1 - C' - I')$ and $(1 - C_1' - I_1')$ are both greater than zero and less than unity. In this case $D > 0$.

As soon as any ambiguities about the sign of D is done away with, it is possible to estimate the direction of the changes in the variables.

The change in the income of the country which experienced the shock is positive, as expected. Equation 19 represents the multiplier with respect to the shift. Ceteris paribus, the greater the aggregate leakage of the second country, being the numerator of the RHS term in Equation 19, the larger the increase in the first country's income consisting of the sum of "foreign repercussions" through increased exports induced by the second country's high propensity to import.

Under the condition that the determinant is positive, the income of the second country will increase, if the marginal propensity to import of the first country is positive. This can be thought of as being the normal case. If M' is negative, Y_1 decreases, the economic reason being the reduc- of induced exports of the second country as a consequence of first country's negative import function.

Equation 23 tells one what will happen to the consumption of domestically produced goods and services of the first country. Under the specified conditions, it increases since by definition

$$C' > M'.$$

Investment rises in country Y for reasonable values of the parameters as combined in Equation 24.

Consumption of domestically produced goods and services rises in country Y_1, too, if the first country's marginal propensity to import is positive. This result apparently works via increases in income which induce increases in consumption of domestic commodities. (Equation 27)

The same can be said about Equation 28 investment expenditures in the second country.

The above results refer to ultimate equilibrium values made up by all induced changes in the variables as a consequence of the autonomous shock. For instance, let the following values be assumed:

$$C' = .85,\ M' = .05,\ I' = .10,\ C_1' = .81,\ M_1' = .01,\ I_1' = .10.$$

Then

$$D = .0095$$

and the income change of the country in which the shock originated,

$$Y_a = 10.5,$$

while the income change in the other country,

$$Y_{1a} = 5.2.$$

$$C^*_a = 1.2, \; I^*_a = 2.10, \; C^*_{1a} = 4.16 \text{ and } I^*_{1a} = 1.57.$$

All expressions whose sign was dealt with are "multipliers" and various terms can be used to refer to them. This however would be pointless, since any marginal relationship is a multiplier and there is no end to the number of such relationships that can be derived from economic models.

The other kind of shock which may be considered is an increase in the average propensity to consume domestically produced goods and services followed by no changes in the average propensities of the other functions. Since such a change is represented by an additive constant in the consumption function, it is evident by induction that similar results to those of a shift in the marginal-efficiency-of-capital schedule hold with respect to the same variables.

This however might sound somewhat strange in an economic sense. Of course, in the mathematical model of the international economic system under consideration there are no functional relationships describing the capacity effect of investment and relating deviations from full-capacity output to investment decisions. The model, being a comparative statics one, gives information only as regards transmissions of income leading to a new equilibrium position.

Since the formal aspect of an increase in the average propensity to consume domestically produced commodities are identical to those of an increase in the marginal efficiency of capital, it is intended to turn next to the effect of a shift occurring in economy Y from domestic goods and services to foreign commodities. This shift can be represented by a negative parameter in the income-identity or a positive parameter in the import function of the country Y, and a positive parameter of equal obsolute value in the income identity of the second country. Let this parameter be μ.

$$Y = C(Y) + I(Y) - M(Y) - \mu + M_1(Y_1) \tag{29}$$

$$Y_1 = C_1(Y_1) + I_1(Y_1) + M(Y) + \mu - M_1(Y_1). \tag{30}$$

Differentiating Equations 29 and 30 with respect to it obtains:

$$Y_\mu = C' Y_\mu + I' Y_\mu - M' Y_\mu + M_1' Y_{1\mu} - 1 \tag{29'}$$

$$Y_{1\mu} = C_1 Y_{1\mu} + I_1' Y_{1\mu} + M' Y_\mu - M_1' Y_{1\mu} + 1 \tag{30'}$$

where

$$Y_\mu = \frac{\partial Y}{\partial \mu}.$$

Rearranging terms, in matrix notation:

$$\begin{bmatrix} 1 - C' - I' + M' & M_1' \\ M' & 1 - C_1' - I_1' + M_1' \end{bmatrix} \begin{bmatrix} Y \\ Y_1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \tag{31}$$

Upon solving for the unknowns Y_μ and Y'_μ by Cramer's Rule:

$$Y_\mu = \frac{C_1' + I_1' - 1 - 2M_1'}{D} \qquad (32)$$

$$Y_{1\mu} = \frac{1 - C' - I' + 2M'}{D} \qquad (33)$$

where again D is the determinant of the matrix of the coefficients of the system 31.

In order to determine the effects on the sectors of the economies, consider again the two sub-systems:

$$C^* = C(Y) - M(Y) - \mu \qquad (34)$$

$$I^* = I(Y) + M_1(Y_1) \qquad (35)$$

Differentiating Equations 34 and 35 with respect to μ, the results derived are:

$$C^*_\mu = (C' - M')Y_\mu - 1 \qquad (34')$$

$$I^*_\mu = I'Y_\mu + M_1' \, Y_{1\mu} . \qquad (35')$$

Substituting into Equations 34' and 35' from Y_μ and $Y_{1\mu}$ from Equations 32 and 33:

$$C^*_\mu = (C' - M')\frac{C_1' + I_1' - 1 - 2M_1'}{D} - 1 \qquad (36)$$

and

$$I^*_{\mu} = I' \frac{C_1' + I_1' - 1 - 2M_1'}{D} + M_1' \frac{1 - C' - I' + 2M'}{D} . \quad (37)$$

In the same way, by differentiation of the other country's sub-system:

$$C_1^* = C_1(Y_1) - M_1(Y_1) \quad (38)$$

$$I_1^* = I_1(Y_1) + M(Y) + \mu \quad (39)$$

with respect to μ it obtains:

$$C^*_{1\mu} = (C_1' - M_1')Y_{1\mu} \quad (38')$$

and

$$I^*_{1\mu} = I_1'Y_{1\mu} + M'Y_{\mu} + 1. \quad (39')$$

Upon substituting for Y_{μ} and $Y_{1\mu}$ from Equations 32 and 33, we have:

$$C^*_{1\mu} = (C_1' - M_1') \frac{1 - C' - I' + 2M'}{D}$$

and

$$I^*_{1\mu} = I_1' \frac{1 - C' - I' + 2M}{D} + M' \frac{C_1' + I_1' - 1 - 2M_1'}{D} + 1. \quad (41)$$

Assuming stability in both countries, D is positive, and Y_μ negative, as expected, and Y'_μ positive.

In other words, income increases in country Y_1 and decreases in country Y.

Assuming, further, positive marginal propensities to import, if the second country is unstable, the determinant D is negative. For country's Y income to increase, the marginal propensity to import of the second country must fulfill the condition:

$$M'_1 > \frac{C'_1 + I'_1 - 1}{2}$$

Under the above circumstances, the second country's income declines, ($Y_{1\mu}$ being negative).

If the unstable country is the first - the marginal propensity to import being positive - , the determinant is negative again, and the unstable country's income increases. What happens in the other country depends on whether

$$M' < \frac{C' + I' - 1}{2} \text{ or } M' > \frac{C' + I' - 1}{2}$$

In the first case Y_1 increases, in the second it declines. If

$$M' = \frac{C' + I' - 1}{2}$$

no changes occur in Y_1.

The consumption of domestically produced commodities declines in the first country, since

$$Y_\mu < 0$$

and

$$C' > M'$$

(Equation 34').

The result is indeterminate in the case of investment in the same country.

In the second country, in the normal case consumption of domestically produced commodities increases and its investment change is indeterminate. Investment, in general, must move in the direction of change in income in either country Y or country Y_1 and is not excluded to move in the same direction in both.

What happens in cases of alternate stability in one country and instability in the other can be deduced in the same way quite easily from the above equations.

Before closing this section, it is intended to consider briefly the effects of a shift in the average propensity to import without changes in the other propensities, which is distinct from the previous case in that the difference between aggregate consumption and imports remains the same after the shift occurs and the shock is expressed as an additive positive constant in the income identity of the second country.

Finally, the transfer-of-capital question is reviewed.

If there is an autonomous increase in aggregate consumption of the first country, without reduction in any other average propensity in the system except the average propensity to save, due to an increase in the demand for foreign goods and services; this autonomous increase can be represented by an addition in the income-determination equation of the second country of a parameter equal to the value of the spontaneous increase in question.

The expression of the first country's income is not going to be affected directly. The multiplier value of the final income equilibrium will incorporate the induced changes in the variables that constitute the first country's income as a consequence of the autonomous and induced changes in the second country's income components. In the above case, unlike the previously considered shocks, one deals with a change in one of the leakages out of the income flow of the first country which affects the injections into the income flow of the other country in a positive manner.

Let this spontaneous shift be w.

The system now becomes:

$$Y + C(Y) + I(Y) + M_1(Y_1) - M(Y) \tag{42}$$

$$Y_1 = C_1(Y_1) + I_1(Y_1) + M(Y) - M_1(Y_1) + w. \tag{43}$$

Differentiating the system with respect to the parameter w obtains in the familiar way:

$$\begin{bmatrix} 1 - C' - I' - M' & - M_1' \\ M' & 1 - C_1' - I_1' + M_1' \end{bmatrix} \begin{bmatrix} Y_w \\ Y_{1w} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{42'}$$

Upon solving first for Y_w and Y'_w, where

$$Y_w = \frac{\partial Y}{\partial w}$$

and

$$Y_{1w} = \frac{\partial Y_1}{\partial w} ,$$

one gets:

$$Y_w = \frac{M_1'}{D} \tag{44}$$

$$Y_{1w} = \frac{1 - C' - I' + M'}{D} . \tag{45}$$

The final equilibrium values of the sectors of the two economies are obtained by differentiating again the two familiar sub-systems with respect to w.

The final solutions are:

$$C^*_W = (C' - M')\,\frac{M'_1}{D} \tag{46}$$

$$I^*_W = \frac{1 - C' + M'}{D}\,M'_1 \tag{47}$$

and

$$C'_{1W} = (C'_1 - M'_1)\,\frac{1 - C' - I' + M'}{D} \tag{48}$$

$$I^*_{1W} = 1 + \frac{1}{D}\left[I'_1\,(1 - C' - I' + M') + M'M'_1\right]. \tag{49}$$

One observes that the first country's income will increase if and only if the marginal propensity to import of the second country is positive, considering the determinant D as positive.

The economic sense that this condition makes is easy to realize. The induced imports of the second country as a consequence of the autonomous increase in its income will cause an increase in the first country's income, which means that the "automatic balancing mechanism" of income effects is analytically equivalent to a stable international economic system.

The second country's income will increase ultimately - that is the primary increase is not going to be counterbalanced by negative induced changes - if and only if the first country is stable, namely, its aggregate propensity to expend on domestic commodities is less than unity, postulating D to be positive.

With respect to each particular sector one observes that in the normal case where the marginal propensities to import are assumed positive and each country separately is assumed stable, increases are bound to occur.

One may turn now to a well-discussed type of shock, the long-run transfer of capital, or income which can be analyzed within the framework already presented. Metzler's ideas, (22), are going to be substantially followed. These can be considered as really definitive under the assumptions established to prevail throughout this paper.

The problem of capital or income transfers was discussed, Angell (2), Haberler (10), Viner (35), Meade (19) on the basis of an economy operating in a "classical" setting.

The most crucial element of such a model was the price movements as explained by a "quantity theory" of money.

Any transfer of capital or income would depress the paying country's price level and inflate the price level of the receiving country. There would follow an export surplus of the balance of trade of the paying country and an import surplus in the trade balance of the receiving country. This difference would constitute the real transfer, on the assumption of price elastic demand for M and price elastic supply of X.

The essential difficulties and deficiencies of this approach were, of course, the questionable validity of the quantity theory of money in view of the institutional framework of the advanced societies in the last two centuries; the dependence on assumptions about foreign elasticities which had to obey the Marshall-Lerner conditions to produce effects in the presence of price level differentials; and the further assumption, granting the previous ones, that the total sum of the foreign lending or the unilateral payment had to be spent in the receiving country to produce its effects in full.

It was only natural that the improvement in analytical methods, brought about by the Keynesian income approach would make it possible for the problem to be solved, i.e. to specify conditions necessary and sufficient to produce an income transfer through foreign lending or unilateral payments. The size of the effect is found to depend on the course of other variables and nothing can be conclusively said without establishing assumptions about them. Specifically, the income determination components, spending on domestic goods and services in the receiving country and the paying country, investment expenditures in either country; and the foreign leakage that operates in either receiving or paying country may lead to an increase or a decrease or may not affect at all the final real transfer in terms of the intended amount of foreign lending or unilateral payment.

It is proposed to examine these questions in some detail. One need only add that all necessary assumptions are postulated again in order to abstract from price effects in general.

The transfer problem is basically a problem in economic policy. It can consequently be examined under various assumptions about the purposeful endeavors of the governments concerned with respect to "internal balance" or domestic employment policy and "external balance" or balance of payments policy, Meade (18). The basic analysis is only slightly modified to account for any such additional information and the most natural premise under which to proceed is to postulate balanced-budget policies in both countries in order to do away with any fiscal policy effects.

It is possible to imagine three likely alternatives relating to primary effects of the transfer process on the levels of income in either or both countries. Specifically, income may be affected directly in both countries or may be affected in the paying country only, or in the receiving one only. One must then examine the ultimate effects consisting of the sum of direct changes plus all induced changes to their limit.

Income is affected in both countries directly, if it is assumed that the paying country disinvests the transferable value and the receiving country invests the same amount. That is to say, national expenditure on investment in the first country decreases while the second country's increases.

Income is affected directly in the paying country only, if the disinvestment of the first country is not followed by investment in the second country, while income is directly affected in the second country only if the investment in the latter as a consequence of the transfer is not followed by disinvestment in the former.

On the basis of the above considerations, one can construct the model of the two-country international economic system to reflect these possibilites.

$$Y = C(Y) + I(Y) + M_1(Y_1) - M(Y) - e - f \qquad (50)$$

$$Y_1 = C\ (Y_1) + I_1(Y_1) + M(Y) - M_1(Y_1) + e + f' \qquad (51)$$

$$B + M_1(Y_1) - M(Y) \qquad (52)$$

In the system represented by Equations 50, 51 and 52 the parameter e stands for the transfer followed by disinvestment in the first country and investment in the second, the parameter f refers to the direct effect on the first country's income only and f' represents the direct effect on the second country's income only. The identity 52 is the balance of trade.

We are primarily interested in the ultimate effects on B of any parameter indicating a transfer, since it is the final difference in the balance of trade resulting from the transfer shock that measures the net amount of any unilateral payment or long-run foreign lending.

From Equations 50 and 51 one obtains solutions for

$$\frac{\partial Y}{\partial e} = Y_e$$

and

$$\frac{\partial Y_1}{\partial e} = Y_{1e}.$$

These are given by the system:

$$\begin{bmatrix} 1 - C' - I' + M' & - M_1' \\ - M' & 1 - C_1' - I_1' + M_1' \end{bmatrix} \begin{bmatrix} Y_e \\ Y_{1e} \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

$$Y_e = \frac{C_1' + I_1' - 1}{D} = - \frac{1 - C_1' - I_1'}{D}$$

$$Y_{1e} = \frac{1 - C' - I'}{D}$$

Substituting these values into

$$\frac{\partial B}{\partial e} = M_1' Y_{1e} - M' Y_e$$

one gets:

$$\frac{\partial B}{\partial e} = \left[M' \ (1 - C_1' - I_1') + M_1' \ (1 - C' - I') \right) \frac{1}{D} \qquad (53)$$

where

$$D = (1 - C' - I' + M') \ (1 - C_1' - 1_1 + M_1) - M' M_1' \ .$$

With the same procedure mutatis mutandis, one finds:

$$\frac{\partial B}{\partial f} = M' \ \frac{1 - C_1' - I_1'}{D} \qquad (54)$$

and

$$\frac{\partial B}{\partial f'} = \frac{M_1'}{D} \ (1 - C' - I') \ , \qquad (55)$$

having substituted into 54 and 55 the values:

$$Y_f = \frac{1 - C_1' - I_1' + M_1'}{D} \ , \ Y_{1f} = - \ \frac{M'}{D}$$

and

$$Y_{f'} = \frac{M_1'}{D} \ , \ Y_{1f'} = \frac{1 - C' - I' + M'}{D}$$

where

$$\frac{\partial Y}{\partial f} = Y_f \, , \frac{\partial Y_1}{\partial f'} = Y_{1f'} \text{ etc.}$$

Assuming that the international economic system is stable - determinant D is positive - one can analyze the effects of the transfer under the possible combinations of the assumptions about the direct effects on each country's expenditures and each country's stability.

In the case where income is affected in both countries directly by the transfer and both countries are stable, income increases in the receiving country and falls in the paying country. The trade balance, upon examining the terms of Equation 53, improves in the paying country but not to the extent of the amount of the transfer. In formal terms,

$$0 < \frac{\partial B}{\partial e} < 1$$

as can easily be seen from comparing numerator and demonator of the R.H.S. expression in 53, assuming the normal case of positive propensities to import.

When the paying country is unstable and the receiving one stable, income declines in both and the trade balance of the paying country increases by more than the amount of transfer, namely

$$\frac{\partial B}{\partial e} > 1.$$

If the receiving country is unstable and the paying one stable, income increases in both and the trade balance increases in it by more than the amount transferred.

In the case where income is affected directly by disinvestment in the paying country only, income falls in both countries and the trade balance of the paying country improves by less than the amount of the transfer,

$$0 < \frac{\partial B}{\partial f} < 1,$$

Equation 54, if both countries are stable.

If the paying country is unstable and the receiving stable, income falls in both but the trade balance of the paying country increases by more than the amount transferred, or

$$\frac{\partial B}{\partial f} > 1.$$

When the paying country is the stable one, income falls in both and the trade balance in it declines by more than the transfer, in other terms,

$$\frac{\partial B}{\partial f} < -1.$$

Finally, in the case where the direct effect of the transfer is felt in the receiving country's income, if both countries are stable, income increases in both, and the trade balance of the paying country increases by less than the transfer. (Analyze the terms of Equation 55). If the paying country is unstable income increases in the paying country, falls in the receiving country and the trade balance declines in the former.

If the receiving country is unstable, income increases in both and the trade balance increases on the paying country by more than the transfer.

We have thus concluded the examination of international transmissions of income through the mechanism of induced changes in the variables. Within the above given framework many connected questions can be easily answered. It is interesting to analyze the effects of different government policies in view of attaining specified goals. This however will not be undertaken here, since it would bring the dission far afield.

EQUILIBRIUM RATE OF GROWTH AND THE INTERNATIONAL ECONOMIC SYSTEM

The determination of an equilibrium rate of growth of an open economy and of the international economic system can be analyzed in terms of the Harrod-Domar approach which, notwithstanding its grave shortcomings, is very suggestive as to the role of the foreign sector, Johnson (15).

The equilibrium condition in an open economy in the static case is that the leakages out of the system equal the offsets, that is $I + X = S + M$, where S is savings, the other symbols having the meaning attached to them throughout the paper.

The dynamisation of this condition is quite easy. Consider

$$(s + m)\, Y_t = X_t + I_t \qquad (1)$$

where s and m stand for the marginal propensity to save, assumed constant, and the marginal propensity to import, respectively, which is also assumed constant.

This condition refers to the demand side of the economy. The completion of the picture, in order to determine the equilibrium rate of growth, requires a capacity equation.

$$Y_t = \frac{I_t}{g} = bI_t, \qquad (2)$$

defining

$$\Delta Y_t = Y_{t+1} - Y_t.$$

Equation 2 relates an increase in capacity income to investment where g is the capital-output requirement and b its reciprocal. Equations 1 and 2 together determine the equilibrium rate of growth. If Equation 1 relates investment required at time t to utilize fully existing capital and meet the rate of exports X_t required to offset the leakage M_t (= mY_t), any increase in capacity output overtime has to be matched by equal increases in aggregate demand. Equation 2 gives the increase in capacity output as a function of investment and one is able to solve for a rate of growth satisfying simultaneously Equations 1 and 2.

The assumption that increases in capacity output depends only on investment is very restrictive, when especially g is assumed constant. In the extension of this growth approach which has been undertaken, it is further postulated that investment utilizes only domestic output, in other words, that domestic productive capacity does not require imported goods. Otherwise Equation 2 should have included a term for the contribution of imports to productive capacity. Again, it is assumed that imports are intended for consumption purposes. Furthermore, it is postulated that negative or positive trade balances are financed by unilateral payments or by short-run capital movements. This assumption is necessary to keep the analysis as simple as possible. All the above assumptions are ordinary in the multiplier analysis.

The definition of the rate of growth is

$$\frac{Y_{t+1} - Y_t}{Y_t}\,.$$

$$\frac{Y_{t+1} - Y_t}{Y_t} = \frac{b\ (s + m)\ I_t}{I_t + X_t} \tag{3}$$

is the rate of growth, which, after some algebraic treatment, becomes:

$$r_t = b\ (s + m - \frac{X_t}{Y_t}) \tag{3''}$$

On the basis of Equation 3 one sees that if the export rate is falling over time the rate of growth increases and vice versa.

By differentiation of Equation 3''

$$\frac{dr}{dt} = b\ \frac{X_t}{Y_t}\ (\frac{\dot{x}}{X_t} - r_t)\,. \tag{4}$$

From Equation 4 it is seen that if the rate of growth of exports is larger than the equilibrium rate of growth the latter is declining over time

$$(\dot{x} = \frac{dx}{dt}).$$

Up to this point the equilibrium rate of growth has only been defined and nothing has been said about its features.

It is not possible to analyze it without an explicit solution to the difference Equation 3'', whose form is:

$$Y_t = \left[b\ (s + m) + 1 \right] Y_{t-1} - X_{t-1} \qquad (3'')$$

It is to be noted again that the stability of the equilibrium path depends on the root of the homogenous part of the above equation and several patterns of behavior are possible? For steady growth

$$m < C' - 1.$$

In a two-country international economy, the interaction of the two equilibrium rates of growth can be investigated in the following manner by replacing X_t by $m_1 Y_{1t}$ and X_{1t} by mY_t:

$$r_t = b\ (s + m - \frac{m_1 Y_{1t}}{Y_t}) \qquad (5)$$

$$r_{1t} = b_1\ (s_1 + m_1 - \frac{mY_t}{Y_{1t}}). \qquad (6)$$

Differentiating Equations 5 and 6 with respect to t, one obtains:

$$\frac{dr}{dt} = \frac{bm_1 \cdot Y_1}{Y} (r - r_1) \tag{5'}$$

and

$$\frac{dr_1}{dt} = \frac{b_1 mY}{Y_1} (r_1 - r) \tag{6'}$$

From Equations 5' and 6' one observes that if the first country's equilibrium rate of growth is larger than that of the second country, the former rate would continually increase and the latter decline. The opposite holds true in the reverse position. The growth rate at the equilibrium path would remain constant, if the two countries grow at the same rate. In terms of the assumptions set up at the beginning of this section, one sees that the above results make economic sense. It was found previously that if the export fraction is falling over time, the equilibrium rate of growth increases and vice versa. Since, on the assumption that the first country's equilibrium rate of growth increases

$$(r > r_1),$$

its export rate falls and so does by definition the second country's import rate. If the second country's equilibrium rate of growth were to remain constant, an increasing proportion of its output would have to be channeled into investment; at the same time this is impossible, because the second country's exports are a continually increasing fraction of its output, hence a decreasing proportion of its output would be available for investment purposes.

The form of the equilibrium path of the international economy can be determined for different values of the roots of the system of difference Equations 5 and 6. One can rewrite the system 5 and 6 as follows:

$$Y_t = \left[b\ (s + m) + 1\right] Y_{t-1} + m_1 Y_{1t-1} \tag{7}$$

$$Y_{1t} = mY_{t-1} + \left[b_1\ (s_1 + m_1) + 1\right] Y_{1t-1}. \tag{8}$$

Define

$$\left[b\ (s + m) + 1\right] = \delta$$

and

$$\left[b_1\ (s_1 + m_1) + 1\right] = \delta_1$$

The above system can be consolidated into the following expression:*

$$Y_t = (\delta + \delta_1)\, Y_{t-1} + (m_1 m - \delta\delta_1)\, Y_{t-2}. \tag{9}$$

Equation 9 is a second-order difference equation and may yield a variety of behavior patterns.

The stability conditions for the above equation are:

$$1 - (\delta + \delta_1) - (mm_1 - \delta\delta_1) \quad 0 \tag{i}$$

---

*The mathematical derivation of Equation 9 is attained through the following steps:

a. Solve for Y_{1t-1} in Equation 7. This solution is:

$$Y_{1t-1} = -\frac{1}{m_1}(Y_t - \delta Y_{t-1})$$

b. Substitute this expression into Equation 8:

$$Y_{1t} = -mY_{t-1} + \delta_1 \left(-\frac{1}{m_1} + Y_t - \delta Y_{t-1}\right)$$

or

$$Y_{1t} = -mY_{t-1} - \frac{\delta_1}{m_1}\left[Y_t - Y_{t-1}\right].$$

c. Reduce the time reference by one period:

$$Y_{1t-1} = -mY_{t-2} - \frac{\delta_1}{m_1}\left[Y_{t-1} - Y_{t-2}\right].$$

d. Substitute this last expression for Y_{1t-1} back in Equation 7. We obtain:

$$Y_t = \delta Y_{t-1} - m_1\left[-mY_{t-2} - \frac{\delta_1}{m_1}(Y_{t-1} - Y_{t-2})\right]$$

and

$$Y_t = (\delta + \delta_1)\, Y_{t-1} + (mm_1 - \delta\delta_1)\, Y_{t-2}$$

which is Equation 9.

$$1 + (mm_1 - \delta\delta_1) > 0 \qquad \text{(ii)}$$

$$1 + (\delta + \delta_1) - (mm_1 - \delta\delta_1) > 0. \qquad \text{(iii)}$$

If one retranslates the above conditions in terms of the original parameters, one observes that they are too complicated to make any intuitive economic sense. One can say however that the system is highly unstable* in view of what seem to be reasonable values for b, s, m and the corresponding parameters of the other country.

The attainment of more realistic results depends naturally on the abandonment of the very rigid assumptions of the Harrod Domar approach on the one hand and on the introduction of new terms into the relationships dealing with capacity output. The mathematical treatment however becomes very complicated and looses occasionally much of the suggestiveness which is a merit of this simple type of models, if all due qualifications are made.

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*Another way to see this rigorously, is to take into account that the expression $(\delta + \delta_1)$ is the trace of the matrix of the coefficient of the system whose absolute value equals the absolute value of the sum of the roots. It is a necessary condition that this value be less than the degree of the system, in order that the latter be stable. Unless $b(s + m) + b'(s' + m') < 0$ this necessary condition is not satisfied in our case. This however is highly unlikely.

A GENERALIZED MODEL OF INTERNATIONAL TRADE

Consider a system in which there are n trading regions or countries. The assumptions will be made again as to abstract from price effects, so that the only new feature of the present chapter is the number of trading units. A first-order system describing the international transactions is not a special case based on the assumption of a one-period lag in each independent variable, since higher order systems can be converted into first order ones by successive redefinitions of the variables. A first-order system will then be considered being the general case of linear dynamic models.

The system (1)

$$
\begin{aligned}
Y_{1t+1} &= E_1(Y_{1t}) + m_{12}(Y_{2t}) + \dots + m_{1n}(Y_{nt}) \\
Y_{2t+1} &= E_2(Y_{2t}) + m_{21}(Y_{1t}) + \dots + m_{2n}(Y_{nt}) \\
&\vdots \\
Y_{nt+1} &= E_n(Y_{nt}) + m_{n1}(Y_{1t}) + \dots + m_{n\,n-1}(Y_{n-1\,t})
\end{aligned}
\tag{1}
$$

describes the simultaneous income determination in all trading regions, where $E_i(Y_{it})$ is defined as the total domestic expenditure of the i^{th} country as a function of its income at time t; $m_{ij}(Y_{jt})$ is defined as the purchases of commodities of country j from country i as a function of the j country's income as time t.

It is assumed that $E_i(Y_{it})$ and $m_{ij}(Y_{jt})$ are linear expressions in the coefficients for all i's and j's so that system (1) can be rewritten as:

$$\begin{aligned}
Y_{1t+1} &= E_1Y_{1t} + m_{12}Y_{2t} + \dots + m_{1n}Y_{nt} \\
Y_{2t+1} &= m_{21}Y_{1t} + E_2Y_{2t} + \dots + m_{2n}Y_{nt} \\
&\vdots \\
Y_{nt+1} &= m_{n1}Y_{1t} + m_{n2}Y_{2t} + \dots + E_nY_{nt}
\end{aligned} \tag{2}$$

Subtracting the elements of the column vector Y_{t+1} from both sides of the system (2) it follows:

$$\begin{aligned}
E_1Y_{1t} - Y_{1t+1} + m_{12}Y_{2t} + \dots + m_{1n}Y_{nt} &= 0 \\
m_{21}Y_{1t} + E_2Y_{2t} - Y_{2t+1} + \dots + m_{2n}Y_{nt} &= 0 \\
&\vdots \\
m_{n1}Y_{1t} + m_{n2}Y_{2t} + \dots + E_nY_{nt} - Y_{nt+1} &= 0
\end{aligned} \tag{3}$$

The characteristic equation of the system (2) is then:

$$\left| M(y) \right| = \begin{vmatrix} E_1-y & m_{12} & m_{13} & \dots & m_{1n} \\ m_{21} & E_2-y & m_{23} & \dots & m_{2n} \\ \vdots & & & & \\ m_{n1} & m_{n2} & m_{n3} & \dots & E_n-y \end{vmatrix} = 0$$

where $|M(y)|$ is the characteristic determinant of the system. Expanding the characteristic determinant we obtain a polynomial equation in y

$$y^n + H_1 y^{n-1} + H_2 y^{n-2} + \dots + H_{n-1} y + H_n = 0$$

where the H_i's are coefficients of which H_1 and H_n have the following important properties:

H_1 is equal to

$$(-1) \sum_{i=1}^{n} yi$$

where the yi's are the n roots of the above polynomial equation; and H_n is equal to

$$(-1)^n \prod_{i=1}^{n} yi.$$

Further, H_1 equals the sum of the elements of the principal diagonal of the matrix $M(0)$ multiplied by (-1); and

$$H_n = |M(0)| \; (-1)^n.$$

In other words, the sum of the roots of the characteristic equation of a first-order linear difference equation system is equal to the trace of the characteristic matrix and the product of the roots is equal to the characteristic determinant of the system.

i. If the system is stable the sum of the roots must be less than n, the degree of the characteristic equation, in absolute value.

ii. If the system is stable, the characteristic determinant $|M(0)|$ must be less than unity in absolute value.

iii. If either the trace or the determinant is negative or zero, the system admits of non-real and/or negative roots.

The above three statements are necessary but not sufficient conditions for stability.

As long as no other specifications are made about the magnitudes of the coefficients of the system, unambiguous results cannot be derived; although the above qualitative taxonomy indicates already that stability of each country in isolation is not a necessary condition for the stability of the international system (proposition i).

A "normal" situation is considered to be that in which the matrix $[M]$ is non-negative. If the m_{ij}'s ≥ 0 for all i's and j's, a sufficient condition for the stability of the system is that the norm of the matrix (defined as the largest columnar sum) be equal to or smaller than unity. If it is equal to unity the system cannot explode. If it is smaller than unity, it converges upon an equilibrium value.

The relation between the dynamic model and the static one can be clearly seen from the following considerations.

In the matrix notation, the dynamic system is

$$IY_{t+1} = EY_t + mY_t + k$$

which has the static solution, for $Y_t = Y_{t+1}$

$$IY_t = EY_t + mY_t + k$$

or

$$(I - E - m)\ Y_t = k,$$

where k is a constant vector. In order for the system to admit of economic interpretation the components of the Y_t vector must be positive. The solution of the dynamic system is given by iteration by:

$$Y_t = (E - m)^t\ Y_o + \left[I + (E - m) + (E - m)^2 + \dots + (E - m)^{t-1}\right]k$$

The system will converge to the equilibrium value

$$y_t = (I - E - m)^{-1}\ k$$

if and only if all the roots or the matrix $\left[E + m\right]$ are less than one in modulus.

Hawkins and Simon have shown in their work on input-output models that positive solutions will attain in the case of a positive matrix if and only if all principal minors of $(I - E - m)$ are positive.

If the elements of the matrix are just nonnegative the positivity of the principal minors above guarantees the nonnegativity of the solutions. A similar proposition was proven by Metzler (23).

The above theorems are in a sense inexact, since, as proven by Solow (33), the decomposability of the nonnegative matrix was not taken into account.

The economic interpretation of decomposability is very important. Consider a square matrix n x n. A collection of elements, m_1, ..., m_n will be called a closed set if

$$m_{pq} = 0$$

for any q in the set and any p not in the set. With respect to the international trade model a closed set is a collection of countries which do not purchase from countries not belonging to the collection.

The relative matrix is said to be indecomposable if there is no closed set other than the set of all elements

$$m_{n1}, \ldots, m_{nn} .$$

It is evident that an expenditure originating in any country or sector of a closed set as above defined will not leak out of the system. In a system with various closed sets, in other words, in a decomposable system an expenditure in one set does not create "derived demand" in another.

Then, an exact theorem about sufficient conditions for stability due to Solow (33) states:

If $(E + m)$ is a non-negative, indecomposable matrix none of whose column sums is greater than one and at least one of whose column sums is less than one, then all the characteristic roots of $(E + m)$ have a modulus less than one, that is

$$I + (E + m) + (E + m)^2 + \ldots$$

converges to

$$(I - E - m)^{-1}.$$

Again, the central point in the economic interpretation of the theorem is that

> "We can tolerate one or more, or even all but one, countries balanced on the stability-instability knife-edge of a unitary marginal propensity to spend a. as long as there is at least one country with a positive marginal to save and b. as long as the trading system is so firmly tied together that each country is intrinsically linked to such a stable country."

The investigation of the effects throughout the system of various shocks originating in one country can be carried out by a "general multiplier" analysis.

A shock <u>a</u> originating in country i causes the following changes from the initial equilibrium position:

$$\frac{dY_i}{da} = \frac{M_{ii}}{M} , \frac{dY_k}{da} = \frac{M_{ik}}{M} \text{ etc.}$$

where M_{ij} is the cofactor of the element in the i^{th} row and j^{th} column of the matrix of the system.

$$\frac{M_{ij}}{M}$$

is a general multiplier measuring the total effect on j^{th} country's income, lying in the normal case between the ordinary (closed economy) and the foreign-made multiplier of a single equation model.

THE THEORY OF INTERNATIONAL TRANSMISSIONS AND EMPIRICAL RESEARCH

It is intended to present in this chapter the models used in three studies (27), (29), (4), on international transmissions with main aim the statistical estimation of the relationships involved. The common feature of these studies is the effort to establish in quantitative terms the relation between foreign trade and the level of domestic economic activity of the various countries making up the international system.

The statistical procedures employed in all three studies are the multiple regression and correlation techniques. A common weakness of the approach used by the authors is the single equation model for each particular relationship.

The Neisser-Modigliani study (27)

This is the most ambitious of the three in terms of the complexity of the models used and the breakdown of countries and commodities.

A summary of the general framework is as follows:

1. The countries are divided into industrial and non-industrial according to the composition of their exports during the interwar period. The countries whose exports of manufactured goods have been more than half of the total

were considered industrial, the others non-industrial.

2. Each import function of the industrial countries involves as independent variables on which imports depend a. income, b. prices, c. net stock change, d. net capital flow, e. food production.

3. The import function of each non-industrial country involves as independent variables: a. exports, b. industrial output, c. prices, d. net stock change, e. net capital flow.

4. The exports of any country depend directly upon a. imports of all other countries, b. prices, c. net stock change, d. net capital flow.

5. The breakdown of commodities is into raw materials, food, and manufactured goods.

The following observations about the relative importance of each independent variable of the models* can be made:

a. Income. In all cases it proved to be the "chief import attracting agent". The same can be said about industrial output whenever it was used instead of income.

b. Prices. The effects of prices could not be reliably estimated in all functions. Not rarely, a price ratio was found to influence exports, but as regards imports the price

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*One should speak of <u>models</u> rather than <u>model</u> since each relationship is independent of any other; in other words, the dependent variable of each equation is determined by the independent or exogenous variables of the same equation. There are no simultaneous relationships.

variable had a demonstrable effect only in the functions of industrial countries. Even in this case, most of these price effects had to be omitted for practical purposes from the overall equation system.

c. Net capital flow. It could not be shown statistically to affect the exports of the various members. As to imports, the distinction of industrial and non-industrial countries is pertinent. An increase in the net flow of capital to an industrial country results in an increase in primary imports associated with rising income. The high correlation between income and net capital movements, being the phenomenon of multicollinearity "eliminated both the necessity and the possibility of using both variables in the explanation of industrial countries imports". As to non-industrial countries, a net capital inflow is usually converted into imports of manufactured goods.

The study covers the interwar period, 1924-1937, roughly.

Tables 25 and 26 (27) reproduced from the study, show numerical estimates of income effect coefficients. (Subscripts refer to countries as follows: 1. U. K.; 2. U. S.; 3. Germany; 4. France; 5. group of Italy, Japan, Sweden, Belgium, Switzerland, Czechoslovakia, Austria)

The data below (Income ratio coefficients (d_1), 1928, 1932, 1935: Dollar change in any industrial member's income associated with a $1 change in any other industrial member's income (both in current prices) assuming no countermeasures regarding the trade balance) refer to the transmission mechanism set in motion by a domestically produced change in real income which, through its influence on imports, affects the incomes of the other countries. The assumption is, apart from constancy of prices, that the determinants of income in any other country j remained unchanged, so that the coefficients given below help to determine the induced income changes in country j.

	$\frac{d_1 Y_j}{d_1 Y_1}$	$\frac{d_1 Y_j}{d_1 Y_2}$	$\frac{d_1 Y_j}{d_1 Y_3}$	$\frac{d_1 Y_j}{d_1 Y_4}$
1928				
j = member 1		.010	.055	.036
2	.141		.146	.053
3	.126	.016		.086
5	.124	.019	.110	
1932				
j = member 1		.008	.038	.028
2	.188		.131	.085
3	.077	.013		.091
5	.128	.013	.088	
1935				
j = member 1		.010	.028	.038
2	.131		.079	.080
3	.096	.023		.147
5	.093	.015	.053	

Income elasticity coefficients (d_1) for 1928, 1932, 1935 (Percentage change in any industrial member's income associated with a $1 change in any other industrial member's income (both in current prices), assuming no countermeasures regarding the trade balance) are given as follows:

	$n(Y_jY_1)$		$n(Y_jY_2)$	$n(Y_jY_3)$	$n(Y_jY_5)$
		1928			
j = member 1	%		.043%	.052%	.041%
2	.034			.033	.025
3	.133		.070		.115
5	.138		.062	.082	
		1932			
j = member 1			.027	.033	.036
2	.057			.036	.030
3	.086		.049		.119
5	.127		.036	.067	
		1935			
j = member 1			.029	.035	.036
2	.046			.034	.028
3	.077		.054		.149
5	.093		.042	.065	

The logic of the Neisser-Modigliani approach is not satisfactory from the standpoint of economic theory, however much it may have facilitated the statistical treatment of the equations. The simultaneity of the relationships in any economy can be neglected only for theoretical purposes, in order that a partial analysis may throw light on the intricacies of each relationship in isolation or ceteris paribus. Wherever though the purpose is that of empirical verification of a model or of policy recommendations, a "realistic" picture of the economy can only be attained through interdependence models. When the question of what variables have been considered by Neisser-Modigliani endogenous and what exogenous, one immediately feels that the only excuse is the formidable difficulty of handling a system of simultaneous dynamic equations suitable for the scope of the study!

There are only two endogenous variables, imports and exports. The exogenous variables are a. income, b. prices, c. food production, and d. net capital export of the primaries countries.

There are two economic problems to be considered on the basis of the above distinction. First, whether there is any dependence among the exogenous variables, and second, whether not only do the import and export variables depend on the exogenous variables, but they influence them, in their turn. (The authors ascertain that the exogenous variables passed the test for statistical independence).

Consider the relation of income and prices. What appears to be the most serious doubt about the alleged independence of income and prices has its ground in the particular historical period chosen for the investigation. The time span 1924-1937 is in general a period of "Keynesian" environment. And it is theoretically acceptable that income changes under resource unemployment leave prices unaffected. But the converse is not true. Quite in the contrary, since resources are unemployed, an autonomous positive price level change is expected to raise income through setting in motion the multiplier-accelerator mechanism. So, there is a dependence of income - a dynamic or lagged dependence - on prices in an underemployment situation.

The relation between income and food production is evidently a positive one and only its practical significance for a particular case may be argued.

There is possibly no reason to expect in the short run a direct interdependence between capital export and prices or good production. The relation between capital movements and income in the non-industrial (where only it is relevant in the model) cannot be unambiguously assessed, because to a great extent it depends on whether the derived demand due to a capital movement into a non-industrial country will leak totally or partially out of the system or not. It is however a strong assumption to make that capital imports and income are independent in a non-industrial country where there is a "presumption of underdevelopment". The authors attempted to meet this difficulty by including a variable consisting of total exports plus industrial output instead of income but it is again evident that exports and industrial output are interrelated with income being just components of it.

The last consideration brings the criticism to the second problem, namely whether the endogenous variables influence or not the exogenous variables. If yes, the exogeneity is lost and the results derived from the model are tenuous.

Now it is evident that imports and exports, or more precisely the balance of trade influences income being a component of it.

The very strong assumption for practical purposes has to be made that any change in the balance of trade is compensated by opposite changes in other income components within the same time period. This has been recognized for particular cases by the authors who modified *ad hoc* their equations. But since it is in the main their purpose to explain exports and imports as the only endogenous variables, the weakness is in general unmitigated. The main theoretical criticism then, is the exogeneity assumption so crucial to the Neisser-Modigliani approach. Only a system of simultaneous equations could meet this criticism, if the model were to include the variables used by the authors of the study.

The Polak study

Polak (29) constructed a model for empirical application where the endogenous variables for the world as a whole are the exports (x), national income (y) and imports (m) of each country. Autonomous investment and price ratios are among the exogenous variables.

Using the subscript i to distinguish countries, the following relationships describe essentially his approach.

All variables are in billions of dollars at constant prices.

$$x_i = \sigma_i' \; x_w \tag{1}$$

where σ_i' is the propensity of the world as a whole to purchase from country i;

$$y_i = \frac{x_i}{\delta_i + \mu_i} \tag{2}$$

where δ_i = (1 - propensity to consume - propensity to invest) and μ_i the propensity to import of i;

$$m_i = \mu_i y_i \tag{3}$$

The international reflection ratio being

$$\rho = \frac{\mu}{\delta + \mu},$$

$$m_i = \rho_i \sigma_i' \, x_w + a_i$$

where a_i is the total effect upon imports of all exogenous variables omitted from the above equations.

Then,

$$\sum_i m_i = x_w \sum_i \rho_i \sigma_i' + \sum_i a_i$$

and since

$$\sum_i m_i = x_w = \sum_i x_i \ ,$$

$$x_w = \frac{\sum_i a_i}{1 - \sum_i \rho_i \sigma_i'} \ .$$

where

$$\frac{1}{1 - \sum_i \rho_i \sigma_i'}$$

can be called the "world multiplier".

Polak has obtained numerical estimates of the coefficients referring roughly to 1924-38. The letters P, M, T attached to countries in the summary of his results as it appears below indicate primaries, manufactures, and totals:

Country		Multiplier			
Czechoslovakia	M 2.0308	2.55		0.32	0.82
Denmark	P 0.0098	1.20		0.73	0.88
France	T 0.0570	1.12		0.25	0.88
Germany	M 0.154	2.48		0.23	0.57
Hungary	P 0.0060	1.80		0.37	0.67
Ireland	P 0.0048				0.47
Italy	T 0.0177				1.17
Netherlands	M 0.0319	1.60		0.48	0.77
Norway	T 0.0069	0.69		0.67	0.46
Sweden	T 0.0243	2.53		0.28	0.71
Switzerland	M 0.0193	1.55		2.36	0.56
United Kindgom	T 0.107	2.80		0.18	0.60
Canada	P 0.0311	2.13		0.32	0.68
China	P 0.0189				0.81
India	P 0.0118				0.67
Japan	T 0.0389	1.98	for twenties	0.17	0.50
		2.22	for thirties	0.10	0.34
Argentina	P 0.0165				0.55
United States	T 0.181	5.36		0.45	0.24

Polak's approach is much closer in spirit to the "Keynesian" models employed in the present essay.

The inclusion of income among the endogenous variables to be explained by the model is theoretically more sound than the Neisser-Modigliani approach. Evidently, however, Polak's study suffers from oversimplification.

The equilibrium level of income is given by the exports multiplied by the foreign trade multiplier.

The fact that investment is autonomous makes the picture very remote from reality, notwithstanding our ignorance of how to explain satisfactorily induced investment. The export functions of his model may be statistically easily manageable, but they hardly reveal any economic notion as for instance a country's import function does.

They are statistical regressions without rigorous economic significance. This makes the concept of an international economic system rather economically artificial. In other words, the linkage of each country with any other is not clearly shown as it would be the case if the concept of world exports were not the beginning analytical step but the final. The aggregation over so many countries deprives the notion of an international system of much substance. Finally since again the time period was characterized by underemployment equilibrium prices could not for any useful purpose be treated as endogenous.

The Beckerman study

Beckerman (4) presents a model of the trade network of a ten-sector international economy.

The following equation set describes the model:

$$m_{i1}x_1 + m_{i2}x_2 + \ldots + m_{in}x_n + e_iA = X_i \quad (i = 1,2,\ldots,n) \qquad (1)$$

where X_j is the total volume of exports of sector j; m_{ij} is the ratio of j's imports from i and $m_{ij}x_j$ stands for the imports of j from i. A is the total volume of imports of sector L and e_i is the proportion from i. It is assumed that sector L's imports are independent of exports.

In this ten-sector model, i runs from 1 to 9; the sectors used are 1. Canada; 2. Dollar Latin America; 3. Sterling Members of O.E.E.C.; 4. Continental Western Europe; 5. Overseas territories of Continental Western Europe; 6. Overseas Sterling Area; 7. Non-Dollar Latin America; 8. Eastern Europe (including China); 9. "Others" representing the world except the above mentioned sectors and the United States. L stands for the United States.

In matrix form the system becomes:

$$(I - M)\ X = A_e \qquad (2)$$

where I is the unit matrix, M the m_{ij} matrix, X the column sector of exports, A the sector representing total United States imports and e is the column sector of e_i.

Furthermore,

$$m_j = \sum_{i=1}^{n,1} m_{ij} \qquad (j = 1,\ldots,n) \tag{3}$$

and

$$m'_j = \sum_{i=1}^{n} m_{ij} \qquad (j = 1,\ldots,n) \tag{4}$$

Letting M_i be the total imports of sector i, we have

$$M_i = m_i X_i \qquad (i = 1,\ldots,n) \tag{5}$$

M_i need not be constant.

The use of the term "propensity to import" as used by the author means

$$\frac{\Delta(m_{ij}x_j)}{\Delta\ X}.$$

The major assumption in this study is that the elasticities of demand for imports with respect to exports from individual regions into other regions have remained fairly constant over the period in consideration.

A sector multiplier, say, for sector i, may be defined as the ratio of the final change in i's total exports to the autonomous change in total U.S. imports.

Let it be denoted by

$$k_i = \frac{\Delta X_i}{\Delta A} .$$

Let K represent the column sector of k_i $(i = 1,\ldots,n)$ and $K^* = (I - M)^{-1}$. Then,

$$(I - M) K = e \tag{6}$$

and

$$K = K^*e \tag{7}$$

The total world trade multiplier ($= \overline{K}$) will then be

$$\overline{K} = i' \; K^*e \tag{8}$$

where i' denotes a unit row of k elements.

The reflection ratio of sector j with respect to sector i is defined as the ratio of the final change in i's exports to an autonomous change in U.S. imports, from sector j alone.

The reflection ratio of sector j with respect to the whole world, j's international reflection ratio, is the ratio of the final change in worlds exports from autonomous increase in U.S. imports from sector j alone.

In terms of general conclusions that the author draws, it is that the reduction of the world trade multiplier between 1938 and 1953 is primarily due to changes in the pattern of trade among non-U.S. sectors. The most important change to this effect has been the increase in the proportion of imports from the United States "insofar as this corresponds to a lower marginal propensity to import from the non-U.S. sectors". The most important consideration to which Beckerman is led by his results is this:

> Suppose it is decided that the most unstable element in the economy is investment. It might then appear legitimate to deduce that, in the interest of maximizing the stability of the level of income though not of maximizing its absolute level, the proportion of investment to total income generating expenditure should be made as low as possible. But at the new level of equilibrium at which savings equals investment (abstracting from foreign trade and government sectors, etc.) savings is now also a much lower proportion of total income than previously. That is, the average propensity to consume is now much larger than previously. If it is then assumed that the rise in the average propensity to consume is accompanied by a similar, though not necessarily exactly equal, rise in the marginal propensity to consume, the multiplier will now be greater than before. Thus, stability may not be increased, the rise of the multiplier offsetting the fall in the absolute size of any given proportionate change in the multiplicand.

The implication of the above quotation for the period to which it refers relating to foreign trade is the following: It was sometimes suggested in the interest of foreign trade stability that the non-U.S. sectors should become as far as possible, independent of the United States market, because it is believed that the most unstable element in world trade is the instability of the United States economy. If this independence is established by the non-U.S. sectors switching exports to other sectors than the United States, in order to preserve dollar balance, imports from the United States have to decline pari passu. But if this decline of the average propensity to import from the United States is accompanied by a similar decline in the marginal propensity to import from the United States, the world multiplier will then increase and consequently stability may not be established, since the reduction in the absolute size of any given proportionate changes in the multiplicand (i.e. the United States imports sector) may be offset by that increase in the multiplier.

From the above summary of Beckerman's empirical work it is immediately evident that the orientation of his study was absolutely conditioned by the largely pseudo-problem of the dollar shortage.

The overwhelming international economic position of the United States of that time caused serious fears as to the adaptability of the European economy to meet the American competition. Since the main analytical tool of this study is the international reflection ratio defined in such a fashion as to manifest the motor role assigned to the American economy, and since the international economic conditions have substantially changed in the subsequent time, both the model and the conclusions therefrom have no real interest for the present.

Some of Beckerman's results as given below show sector multipliers, reflection ratios for sterling member countries and reflection ratios for the international system of his model, respectively.

Sector multipliers

	Sector	1938	1948	1953
1.	Canada	.42	.45	.40
2.	Dollar Latin America	.33	.46	.37
3.	Sterling members of O.E.E.C.	1.15	1.02	.75
4.	Continental W. Europe	2.92	1.47	1.86
5.	Overseas territories of C. W. Europe	.29	.26	.23
6.	Overseas sterling area	1.24	1.10	.92
7.	Non-dollar Latin America	.44	.52	.33
8.	Eastern Europe	.57	.28	.13
9.	Others	1.10	.60	.53
10.	Total (= World Trade Multiplier)	8.48	6.14	5.52

Reflection ratios for sterling
member countries

Sector from which U.S. purchases extra dollar of imports	Resultant final increase in total exports of sterling member countries
Sterling member countries	2.14
Others	1.25
Overseas sterling area	1.24
Eastern Europe	1.13
Continental W. Europe	1.08
Overseas territories of C. W. Europe	1.07
Non-dollar Latin America	0.52
Canada	0.34
Dollar Latin America	0.30

Sector	Reflection ratios 1938	1943	1953
1. Canada	4.12	2.84	2.74
2. Latin America (Dollar)	3.72	2.56	3.01
3. Sterling member countries of O.E.E.C.	13.61	9.82	8.49
4. Continental W. Europe	10.32	12.26	8.51
5. Overseas territories of C. W. Europe	8.16	11.24	9.12
6. Overseas sterling area	10.47	9.53	7.52
7. Non-dollar Latin America	8.36	5.36	4.82
8. Eastern Europe	8.71	9.12	8.79
9. Others	11.36	9.25	10.51

CONCLUSIONS

The present essay was an attempt to discuss systematically the income effect in international trade. The discussion assumed the "Keynesian" assumptions and employed the approach and concepts of the post-Keynesian literature of the early fifties' especially. The multiplier and accelerator mechanism has been the main tool of the analysis. Stability conditions were examined and the influence of the marginal propensity to import was seen to be stabilizing under a *ceteris paribus* assumption. The dynamic version of a simple dynamic international trade model was an extension of the Harrod-Domar inquiries into the equilibrium path of income. The international system was a tight one, where the income path of one country was a function of the income path of the other. It was established that the case of instability of one country (case where the aggregate propensity to expend on domestically produced goods and services is larger than unity) can be absorbed into a system of international trade exhibiting stability because of another country's low propensities.

A general international system, where again income was the only variable which attracts imports, was set up and the analysis was mainly confined to qualitative stability propositions. A striking formal analogy Solow (33) between that international system and the input-output analysis may have been noticed.

A system of linear equations

$$y_i = a_{ij}y_j + b_i \qquad (i, j = 1,\ldots,n)$$

is the form of both models in their static version. The interpretation in the input-output case is the following: y_j is the level of production of commodity j, a_{ij} is the technical coefficient of commodity j with respect to commodity i or the per unit of j commodity input requirement of commodity i, b_i is some final (enogenous) demand. The difference in the economic interpretation of the symbols necessitates however somewhat different restrictions of the coefficients in the two models. The input-output matrix of coefficients is restricted to a non-negative matrix, while this is only the "normal" case in the international trade model. It is conceivable that there are countries selling inferior commodities with respect to another country's income in a large proportion, so that the a_{ij} in some particular case (the marginal propensity of country j to import from country i) may be negative. It is to be noted that the international trade model is from a theoretical point of view in a sense less satisfactory than the input-output model. While the latter aims at presenting an instantaneous "technological" picture of the interindustry transactions, the international trade model aims at explaining the network of international trade where the crucial variable is the income of each country. As such it is strictly applicable in an underemployment situation but still even in this case it is more difficult to maintain the assumption of constancy of the relative prices, especially when the theoretical model is intended for practical application. For the price variables which are relevant include such components as tariffs, transportation costs etc. which can hardly be assumed to change proportionately.

The empirical studies which were summarized are a fair sample of the application of the income-effect approach to international trade. The greatest merit of the "Keynesian" models - apart from the theoretical insights they have offered - is that they can be statistically relatively easily managed. Whether they represent the most fruitful approach available, it is to be decided on the general economic atmosphere of the time period under investigation. After 1955, since the world economy operates at a full employment level and the problem is the shortage of resources in many areas, any model intended for empirical research would be crucially deficient without inclusion of price-effects.

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